Framework for Aleatory Variability and Epistemic Uncertainty for Simulations-Based Ground-Motion Characterization for a Level of Simplification

Irene Liou[1] and Norm Abrahamson[1], [1]University of California, Davis

Abstract

As simulations gain wider use as ground-motion models (GMMs) in Probabilistic Seismic Hazard Analysis (PSHA), the aleatory and epistemic components, defining a level of simplification for consistent methodologies across all events, need to be quantified for modern PSHA applications for critical infrastructure. We use an aleatory and epistemic framework to classify the uncertainty components in simulations, which are often assumed to have zero method aleatory variability, despite modeling simplifications, and are typically missing sources of epistemic uncertainty.

Aleatory Variability in Physics-based GMs?

- In deterministic simulations, theoretically $\sigma_{al-Method}$=0; however in practice, they have fixed rules or assumptions for scaling parameters:
  - For example, in Graves and Pitarka (2010) (GP2010) rules of $V_{rup} = 0.56V_s$ for z < 5 km and $V_{rup} = 0.8V_s$ for z > 5 km contribute to $\sigma_{al-Method}$, but is often not quantified.
- If the input parameter is optimized for each specific event it minimizes the misfit with observations.
- Because the value of an optimized parameter is not known for future earthquakes, there are variable realizations for future events, leading to $\sigma_{al-Param}$.

GP2010 Source Terms

Effect	*Single rule for all scenarios**	Scenario specific rules+
Average V_{rup} (e.g., $0.8V_s$)	✓	
Shape of source-time function (e.g., Liu et al. 2006)	✓	
Hypocenter location		✓
Slip distribution (amplitude and rake)		✓

**** Fixed rules or simplifications across different events lead to $\sigma_{al-Method}$***
+ Optimized inputs lead to $\sigma_{al-Param}$ and are randomized for future events

Aleatory and Epistemic Framework

Logic Tree

Type	Method	Parametric
Aleatory Variability $\sigma_{al-Comb}$ $\sigma_{Comb} = \sqrt{\sigma_{Mod}{}^2 + \sigma_{Param}{}^2}$ 3-D Sim: 0.53 NGAW-2 GMMs: 0.7	Method Aleatory Variability $\sigma_{al-Method}$ 3-D Sim SF: 0.25-0.35 [vii] NGAW-2 GMMs: 0.7 [iv]	Parametric Aleatory Variability $\sigma_{al-Param}$ 3-D Sim LA: 0.44 [viii] NGAW-2 GMMs: 0.0
Epistemic Uncertainty of the Median $\sigma_{epi-\mu, Comb}$ $\sigma_{\mu, Comb} = \sqrt{\sigma_{\mu, Method}{}^2 + \sigma_{\mu, Param}{}^2}$ 3-D Sim: 0.51 NGAW-2 GMMs: 0.16	Method Epistemic Uncertainty of the Median $\sigma_{epi-\mu, Method}$ 1-D Sim (3 BBP methods): 0.20 [ii] 3-D Sim SF (1 method): 0.06 [vii] NGAW-2 GMMs: 0.16[i,v]	Parametric Epistemic Uncertainty of the Median $\sigma_{epi-\mu, Param}$ 3-D Sim SF: 0.43-0.49 [vii] NGAW-2 GMMs: 0.0
Epistemic Uncertainty of the Aleatory Variability $\sigma_{epi-\sigma, Comb}$ $\sigma_{\sigma,Comb} = \sqrt{\sigma_{\sigma, Method}{}^2 + \sigma_{\sigma, Param}{}^2}$ 3-D Sim: $\pm$0.05 NGAW-2 GMMs: $\pm$0.03	Method Epistemic Uncertainty of the Aleatory Variability $\sigma_{epi-\sigma, Method}$ 3-D Sim SF: $\pm$ 0.04 [vii] NGAW-2 GMMs: $\pm$ 0.03 [i]	Parametric Epistemic Uncertainty of the Aleatory Variability $\sigma_{epi-\sigma, Param}$ 3-D Sim LA: $\pm$ 0.03 [viii] NGAW-2 GMMs: 0.0

Framework example values for T = 3 seconds; Standard deviations in LN units

Definitions

Aleatory Variability	• **Unmodeled** physical effects in the hazard are treated as randomness in the model • Reduce through the selection of a more complex model
Epistemic Uncertainty	• **Modeled** physical effects in the hazard with scientific uncertainty • Reduce through additional data collection and improved modeling
Method component (Numerical/ Statistical)	• Method aleatory is unexplainable variation from unmodeled effects (simplified fixed terms) comparing with observations • Method epistemic is scientific uncertainty of the center and range of the modeled effects in the numerical or statistical formulation
Parametric component (Input)	• Explainable variability calculated through forward realizations • Needed when parameter is an input to the simulations but not explicitly modeled in the hazard calculation
Hazard equation	$Haz(IM > z) = \sum_{i=1}^{Nsource} N_i(M_{min}) \int_{Mmin}^{Mmax} \int_R f_{M_i}(M) f_{R_i}(R,M) P(IM > z \mid M,R,S)\, dR\, dM$

Missing Epistemic in 3-D Simulations

- Important sources epistemic uncertainty in 3-D simulations :
  - How well is the 3-D velocity model (VM) is constrained by the available geophysical data?
  - How well is the rupture generation methodology is constrained based on past earthquakes?
- Multiple 3-D VMs and rupture generators are needed to capture $\sigma_{epi-\mu, Param}$ but are often missing.

3-D Simulation Example (T=3s)

Misfit between San Francisco (SF) simulations using point source representations with one 3-D VM and 10 moderate SF earthquakes has a standard deviation of 0.55 LN units. Options of how to partition the variance into limitations of the method and uncertainty of the 3-D VM:

- **Method 1**: Categorizing 0.55 as $\sigma_{al-Method}$ treats the VM as a fixed part of the model and treats its errors as apparent aleatory variability. However, VM uncertainty is not zero and the $\sigma_{epi-\mu, Param}$ is missing.
- **Method 2:** Assume the simulation is method is correct despite simplifications. Categorize 0.55 as $\sigma_{epi-\mu, Param}$, which makes the fractiles larger but are reducible as data is collected.
- **Method 3:** Split the 0.55 between $\sigma_{al-Method}$ and $\sigma_{epi-\mu, Param}$. Though the exact partition is unknown, percentages can be assumed (i.e., 30% of variance from source model simplifications in $\sigma_{al-Method}$ (0.30); 70% of variance from VM inputs in $\sigma_{epi-\mu, Param}$ (0.46)).

Conclusion

Use of simulations reduces aleatory variability but increases epistemic uncertainty. An important missing source of epistemic uncertainty in simulations is the 3-D VM. Modelers often assume that the simulation method is completely deterministic ($\sigma_{al-Method} = 0$) and that the only source of misfits is from incorrect inputs. Putting the misfit as $\sigma_{epi-\mu, Param}$ maps aleatory variability from the simplification in the simulation method into epistemic uncertainty. Misfits are more realistically a combination of source model simplifications and scientific uncertainty in the VM. Ideally, there are multiple VMs to capture the epistemic uncertainty.

Citations

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