Geo-INQUIRE

ETRiS - Geo-INQUIRE online training course, First Day: November 6, 2023
Empirical fragility and vulnerability curves for risk analysis (VA2-35-1)

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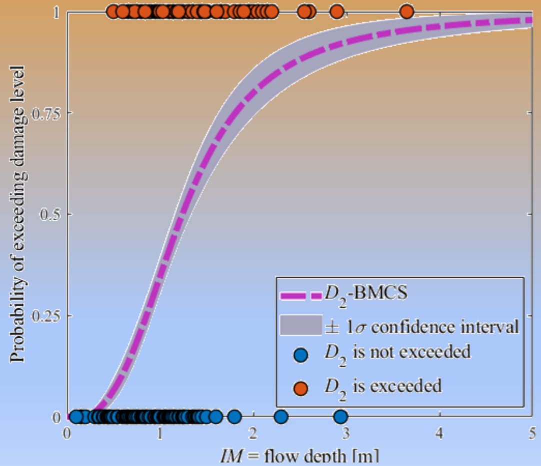

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INSTITUTE FOR RISK AND DISASTER REDUCTION (IRDR)

European Tsunami Risk Service (ETRiS)
Candidate Thematic Core Service (cTCS-Tsu)

EPOS
EUROPEAN PLATE OBSERVING SYSTEM

Agenda: First Day, November 6th

- Introduction to Geo-INQUIRE
- Introduction to EPOS, Tsunami cTCS, and ETRiS
- The forward probabilistic framework
- Damage scales
- Definition of fragility function
- The fragility function for a class of assets
- The definition of vulnerability function
- Empirical fragility curves
- Empirical fragility assessment using GLM
- Bayesian model class selection
- Examples

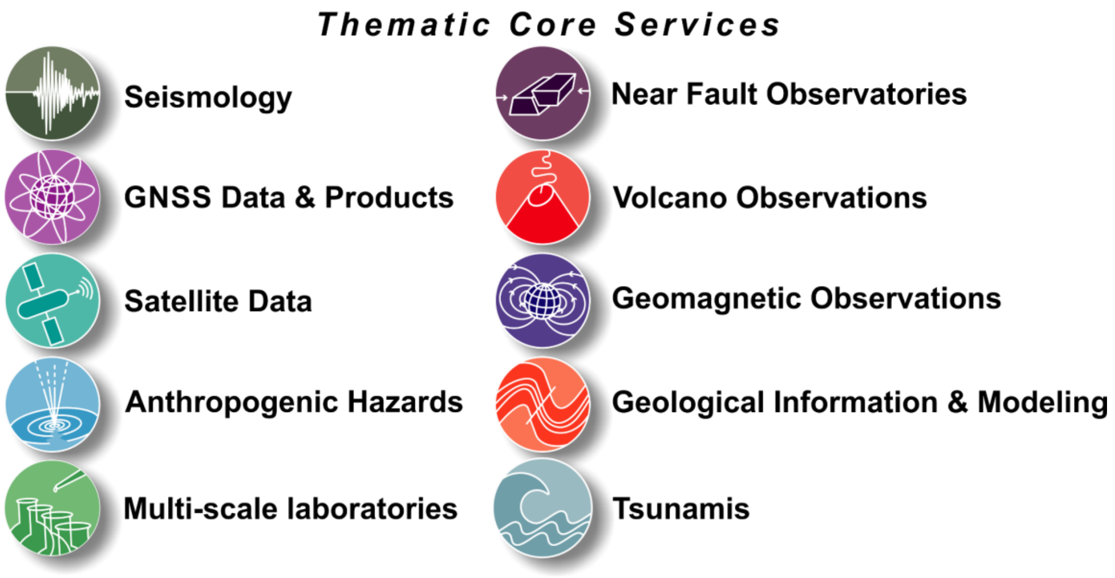
Geo-INQUIRE Project (https://www.geo-inquire.eu/)

- **Geo-INQUIRE** will provide and enhance access to selected key data, products, and services, enabling the dynamic processes within the geosphere to be monitored and modelled at new levels of spatial and temporal detail and precision.

- **Geo-INQUIRE** benefits from a unique partnership of 51 partners consisting of major national research institutes, universities, national geological surveys, and European consortia.

- **Geo-INQUIRE** aims to enhance the scientific community's awareness of the available assets by carrying out **dedicated training programs** to ensure the optimal use of the services, to support access, and to liaise with scientific users to understand how to continuously adapt the services to meet their needs.

European Plate Observing System (EPOS)

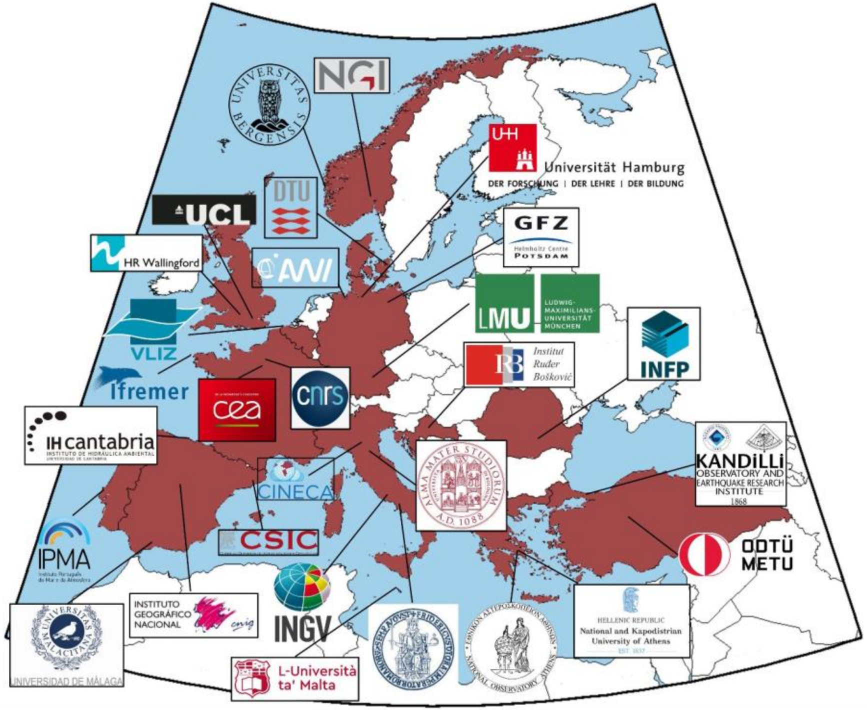


EPOS, the *European Plate Observing System*, is a multidisciplinary, distributed research infrastructure that facilitates the integrated use of data, data products, and facilities from the solid Earth science community in Europe.

Candidate Thematic Core Service Tsunami (cTCS Tsunami)

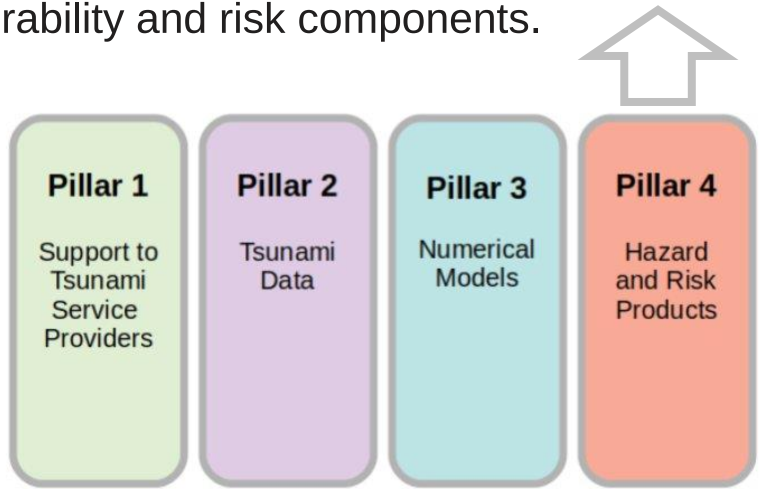
cTCS Tsunami aims to establish sustainable and harmonized services for Tsunami Science and Tsunami Risk Reduction and to coordinate the provision of access to - and interaction with - data, products, software, workflows, and other services on a European level and beyond.

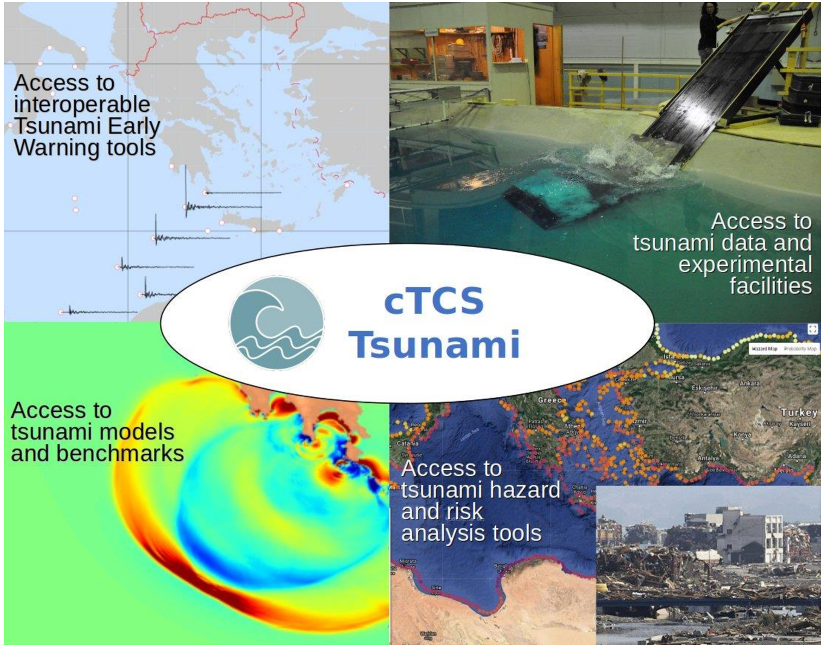
Currently 30 partner institutions across Europe from 14 countries



The European Tsunami Risk Service (ETRiS)

The European Tsunami Risk Service (ETRiS) is a part of the cTCS Tsunami and is integrated into the data portal of EPOS. ETRiS aims to provide virtual access to Data, Data products, Software, and Services for tsunami vulnerability and risk components.





What is the idea behind ETRiS?

- To provide researchers with tools that enable them to do research related to probabilistic tsunami risk assessment (PTRA).

- To post-process raw data in a harmonized way to produce data products.

- To provide open access also to the software used to post-process data.

- To provide access to data products that are useful for multi-hazard and multi-risk applications.

- To provide training activities on how to use the codes to process data and how to use then for PTRA.

A Forward Probabilistic Framework (PTRA)

Hazard

$\lambda(IM > im)$

IM ***(Intensity Measure)***

e.g., flow depth, momentum flux

Vulnerability

$P_{DS|IM}(DS_i \mid im)$

Fragility Function

DM ***Damage Measure***

e.g., damage states

$G_{DV|DS}(dv \mid DS)$

Consequence Function

DV ***Decision Variable***

e.g., fatalities, loss

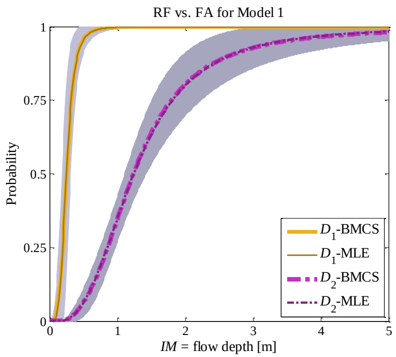
Risk

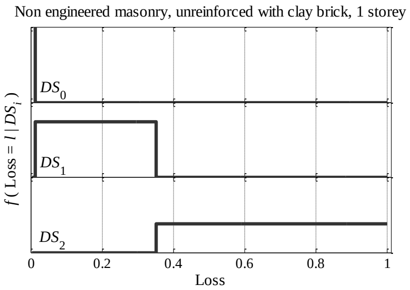
$\lambda(DV > dv)$

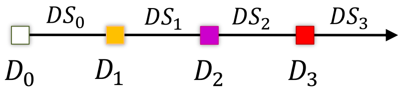
Risk Metrics

e.g., AAL, LEC

Behrens, J., Løvholt, F., Jalayer, F., Lorito, S., Salgado-Gálvez, M.A., Sørensen, M., Abadie, S., Aguirre-Ayerbe, I., Aniel-Quiroga, I., Babeyko, A. and Baiguera, M., 2021. Probabilistic tsunami hazard and risk analysis: A review of research gaps. *Frontiers in Earth Science*, *9*, p.628772.







Decision variable

- The decision variable DV is the generic variable used to describe risk. It is by fact a “metric” of risk such as the number of fatalities, the total economic loss, the annual expected loss, etc.

Exposed Asset

- The procedures we discuss today, apply in general to any asset exposed to risk, a building, a class of buildings, an infrastructure, a class of infrastructure.

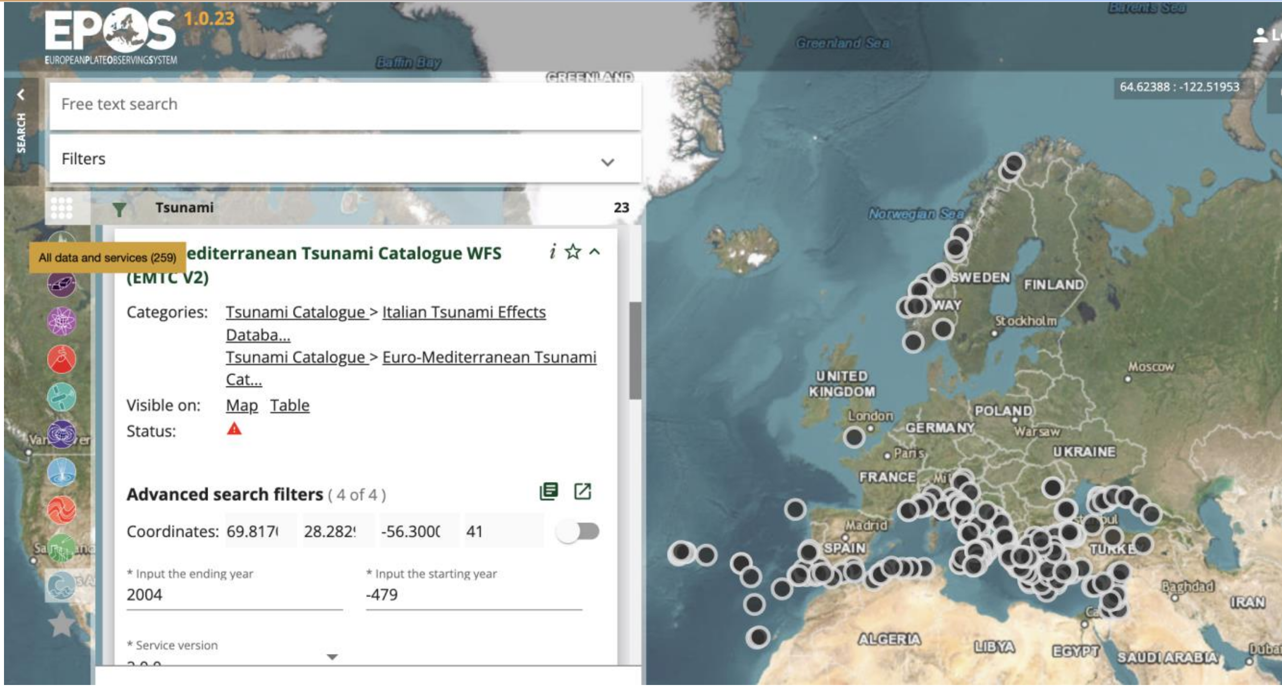
- In certain cases, it can even apply to population at risk.

- Therefore, we use herein, the generic term “exposed asset” to refer to persons and things exposed to risk.

The intensity measure

- Tsunami intensity measure is a scalar or vector that plays the role of interface variable between hazard and vulnerability (e.g., wave amplitude, flow depth, current velocity, momentum flux, or maximum inundation height).

Tsunami Impact and Consequence Datasets



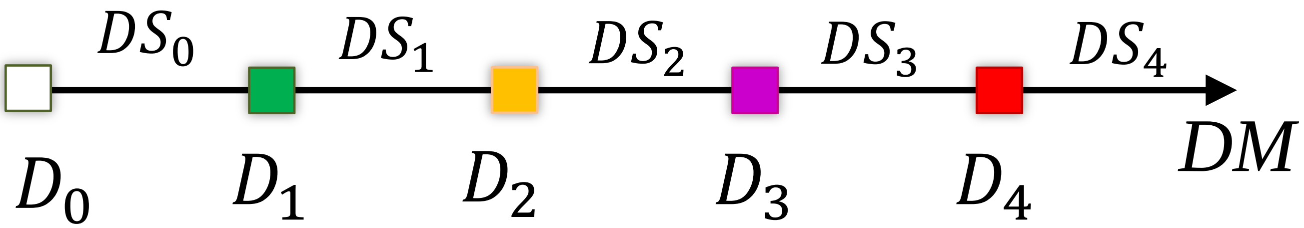
- Euro-Mediterranean Tsunami Catalogue WFS (EMTC V2)
- Italian Tsunami Effects Database (ITED, https://www.ics-c.epos-eu.org/)
- Limited datasets in (csv format) are available on the site of ETRiS.

Tsunami Impact and Consequence Datasets (continued)

- The Japanese Ministry of Land, Infrastructure, and Transport and Tourism (MLIT)
- https://www.mlit.go.jp/toshi/toshi-hukkou-arkaibu.html

Damage scale

- Damage scale is the ensemble of *mutually exclusive ad collectively exhaustive (MECE)* damage states used to described the *whole* range of possibilities in terms of damage for an exposed asset.



Mutually Exclusive and Collectively Exhaustive Damage States

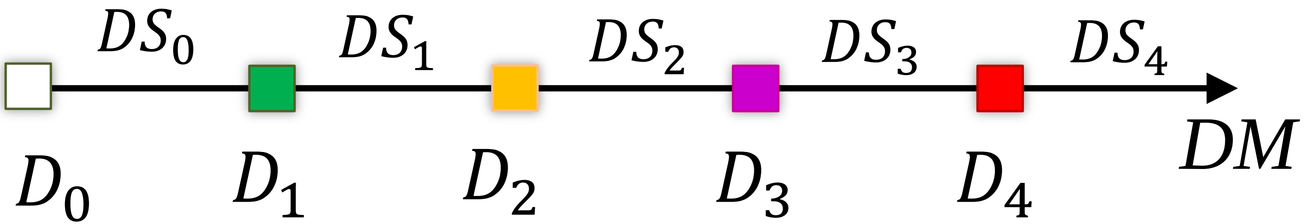
Damage states $\{DS_0, DS_1, \dots, DS_{N_{DS}}\}$ are mutually exclusive and collectively exhaustive (MECE) if an only if

- $P\left(DS_i \cdot DS_j \middle| IM\right) = 0$ (if $i \neq j$, $j = 0{:}\, N_{DS}$)
- $\sum_{j=0}^{N_{DS}} P\left(DS_j \middle| IM\right) = 1$;

Damage Levels and Damage States

Damage levels D_j are the thresholds that separate the different MECE damage states DS_j.

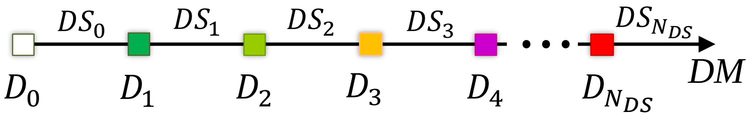
$$DS_j \equiv \left(D \geq D_j\right) \cdot \left(D < D_{j+1}\right)$$



Damage Scales: examples

Damage Level		Damage level description
D_0	None	no damage
D_1	Light	non-structural damage
D_2	Minor	significant non-structural damage, minor structural damage
D_3	Moderate	significant structural and non-structural damage
D_4	Severe	irreparable structural damage, will require demolition
D_5	Collapse	complete structural collapse
		South Pacific 2009 Reese et al. 2011

Damage Level		Damage level description
D_0	None	no damage
D_1	Repairable	Partial damage, repairable
D_2	Unrepairable	Partial damage, unrepairable
D_3	Complete	Complete structural collapse
		Sulawesi 2018 Paulik et al. 2019



Reese, S., Bradley, B. A., Bind, J., Smart, G., Power, W., & Sturman, J. (2011). Empirical building fragilities from observed damage in the 2009 South Pacific tsunami. *Earth-Science Reviews*, *107*(1-2), 156-173.

Paulik, R., Gusman, A., Williams, J. H., Pratama, G. M., Lin, S. L., Prawirabhakti, A., ... & Suwarni, N. W. I. (2019). Tsunami hazard and built environment damage observations from Palu City after the September 28 2018 Sulawesi earthquake and tsunami. *Pure and Applied Geophysics*, *176*(8), 3305-3321.

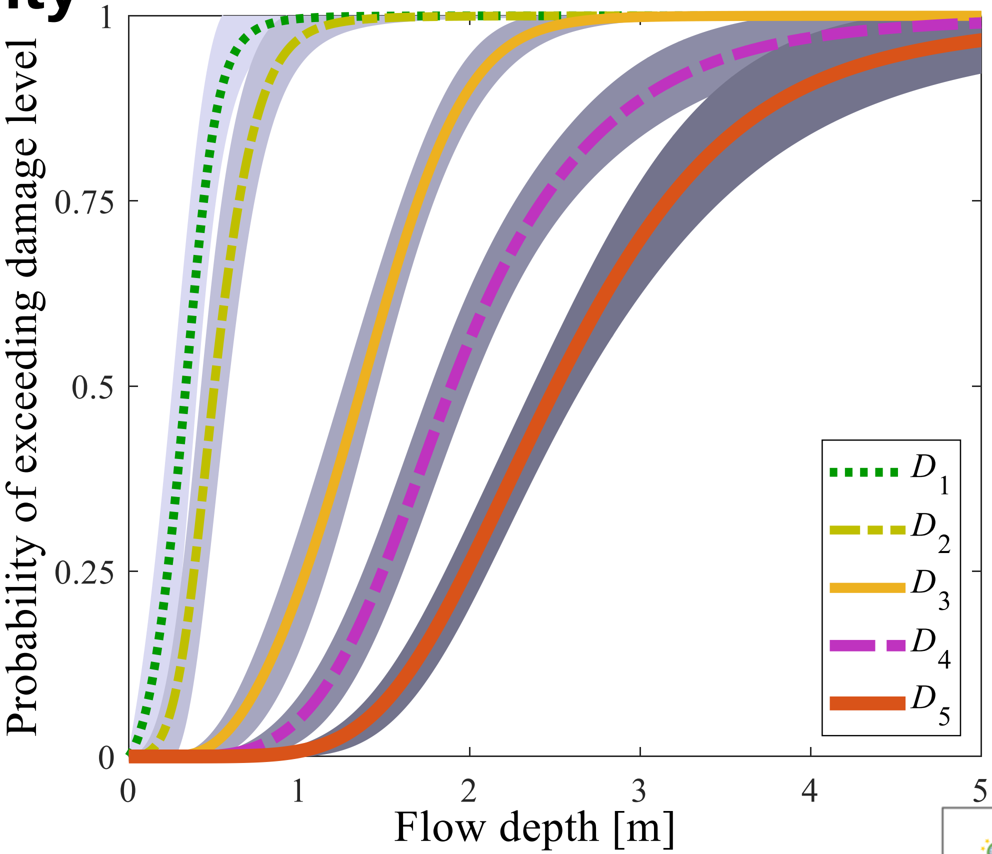
ETRiS: Data Products for Tsunami Risk Assessment

- This page provides access to tsunami vulnerability and risk data products. These data products include various PTRA components such as damage scales, fragility curves, consequence functions, and vulnerability curves. The taxonomy used for labelling elements exposed to risk is GED4ALL. https://eurotsunamirisk.org/dataproducts/
- Examples of Damage Scales in Literature

The definition of the fragility

- In the context of risk assessment at at regional level, the fragility curve is defined as the probability of exceeding a specific damage level as a function of the intensity measure.

$$P(D > D_i | IM = im)$$



The (alternative) definition of the fragility

- The fragility curve can also be defined as the cumulative distribution function (CDF) of the intensity measure value corresponding to the threshold of the damage level:

$$P(D > D_i | IM = im) = P(IM^{D_i} \leq im) = F_{IM^{D_i}}(im)$$

The underlying assumptions

There are some implicit assumptions in the definition of fragility:

- The fragility function is a filter function applied to a homogenous Poisson Process to filter a certain type of event (e.g., those earthquakes which lead to exceeding a certain damage level, or those tsunami events which lead to exceeding a certain damage level).
- It is meaningful for a **single system**. It is assumed that with each new event of interest (from the “mother” Poisson Process), the system will be “renewed” back to its intact state (D_0).

Question: How can we define the concept of fragility for a building class?

The concept of fragility curve for a class

- The fragility curve for a class can be derived by assuming that the portfolio of buildings in a class is replaced by an "average" representative building.

- The dispersion in the class fragility curve, in theory, should consider both the (1) variability in the different types of events (e.g., tsunamis, earthquakes) given the intensity measure; (2) the building-to-building variability within the class.

Empirical Fragility

- Empirical fragility curves are models derived from pairs of observed damage and intensity data for buildings and infrastructure, usually collected, acquired and even partially simulated in the aftermath of disastrous events.

- This implies that damage data are obtained for a group of spatially-distributed surveyed exposed assets.

The empirical fragility curves for a class

- The fragility curve for a class can be derived by assuming that the portfolio of surveyed buildings represent a “class” or are subdivided into more classes and that each class is replaced by an “average” representative structural model.

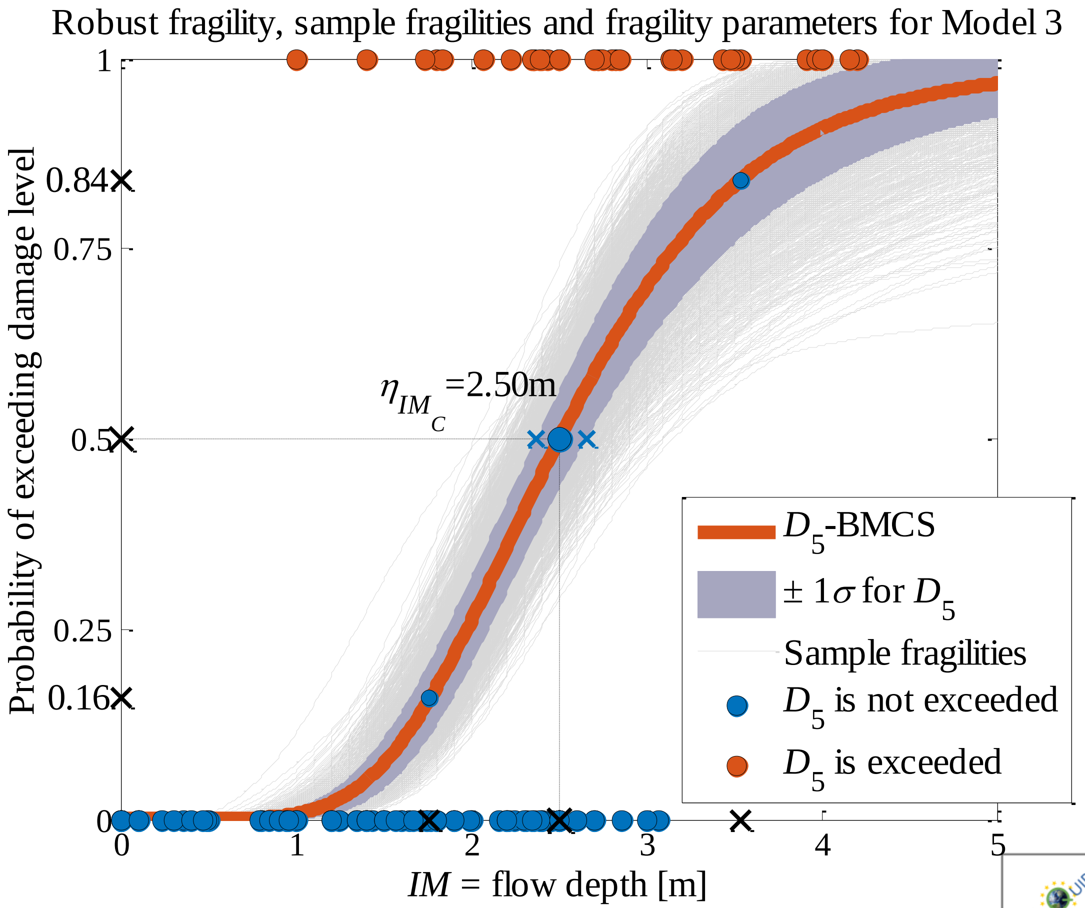
The empirical fragility curves for a class

- This implies a sort of “spatial ergodicity”; that is, the surveyed damage data can be considered as different realizations of the average structural model to the event of interest.

- One consequence of such assumption is that the building-to-building variability is going to be considered in “disguise” and as a contribution to fragility curve dispersion.

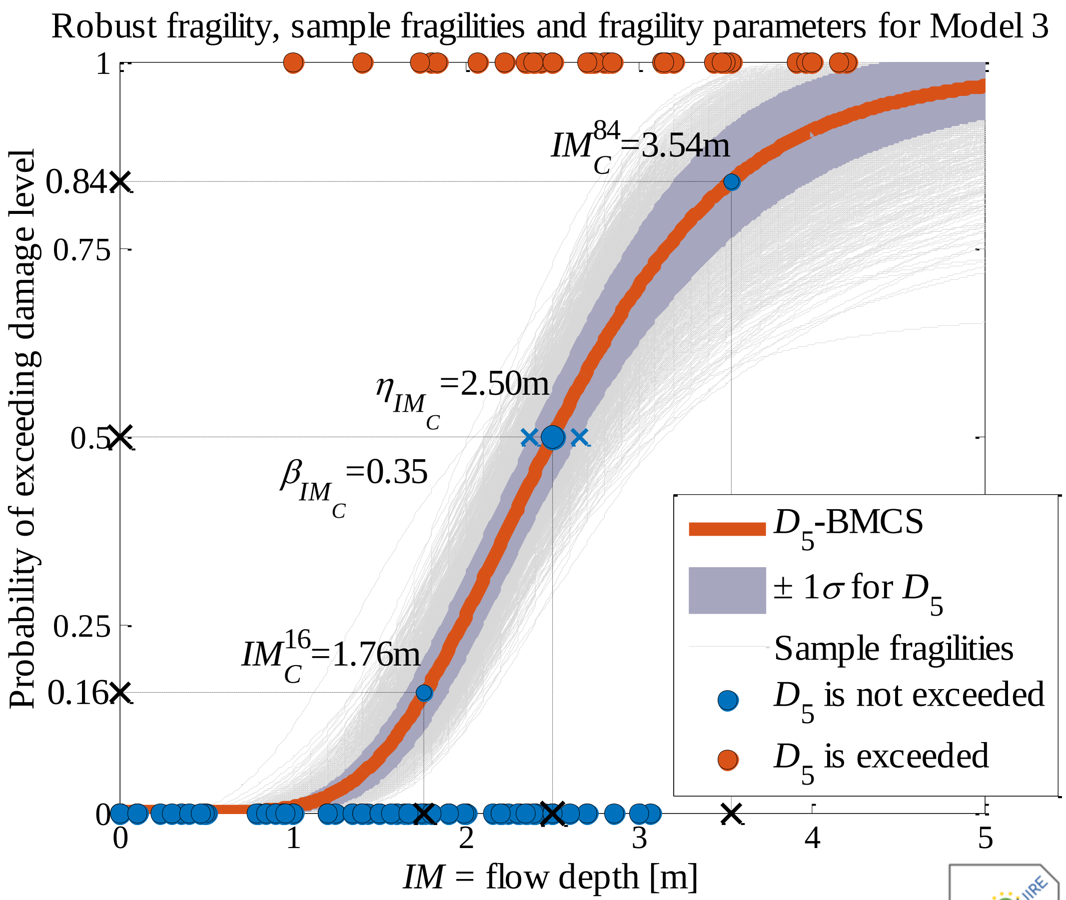
Equivalent Lognormal Statistics for Fragility Curve

- The median intensity, η_{IM_C}, for a given damage level, is calculated as the *IM* corresponding to 50% probability on the fragility curve.



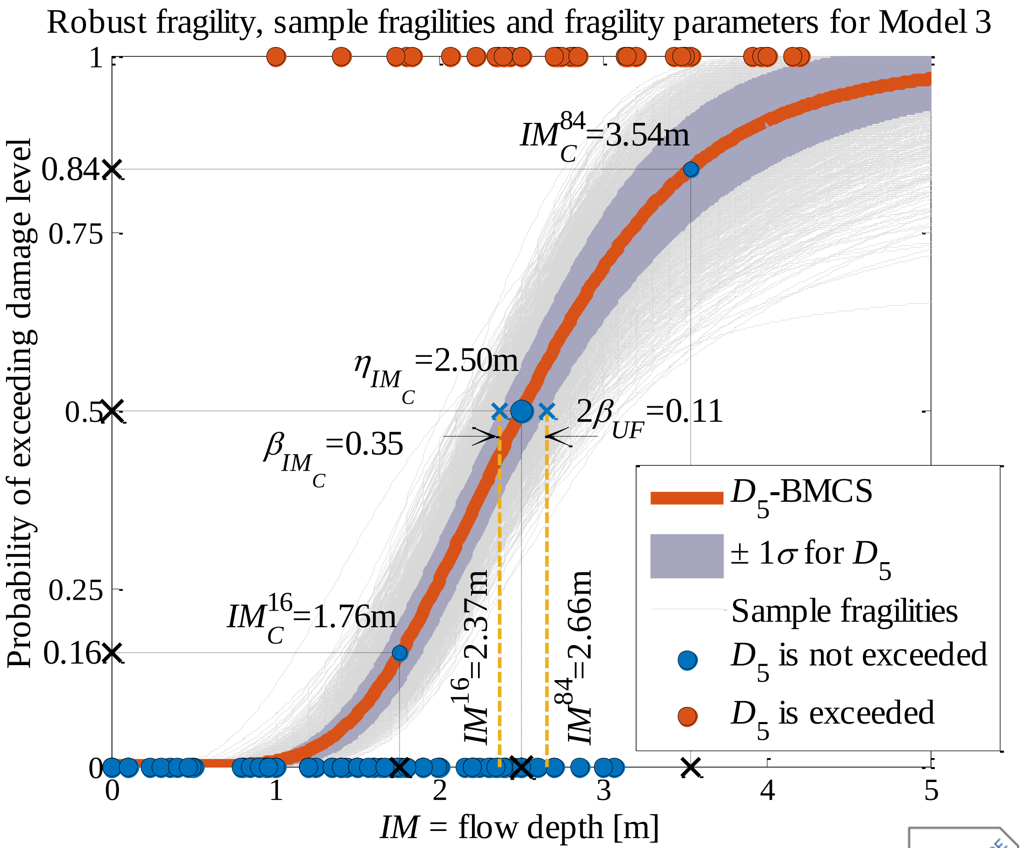
Equivalent Lognormal Statistics for Fragility Curve

- The logarithmic standard deviation (dispersion) of the equivalent lognormal fragility curve, β_{IM_C}, is estimated as as $\beta_{IM_C} = 0.50 \times \ln\left(IM_C^{84}/IM_C^{16}\right)$.



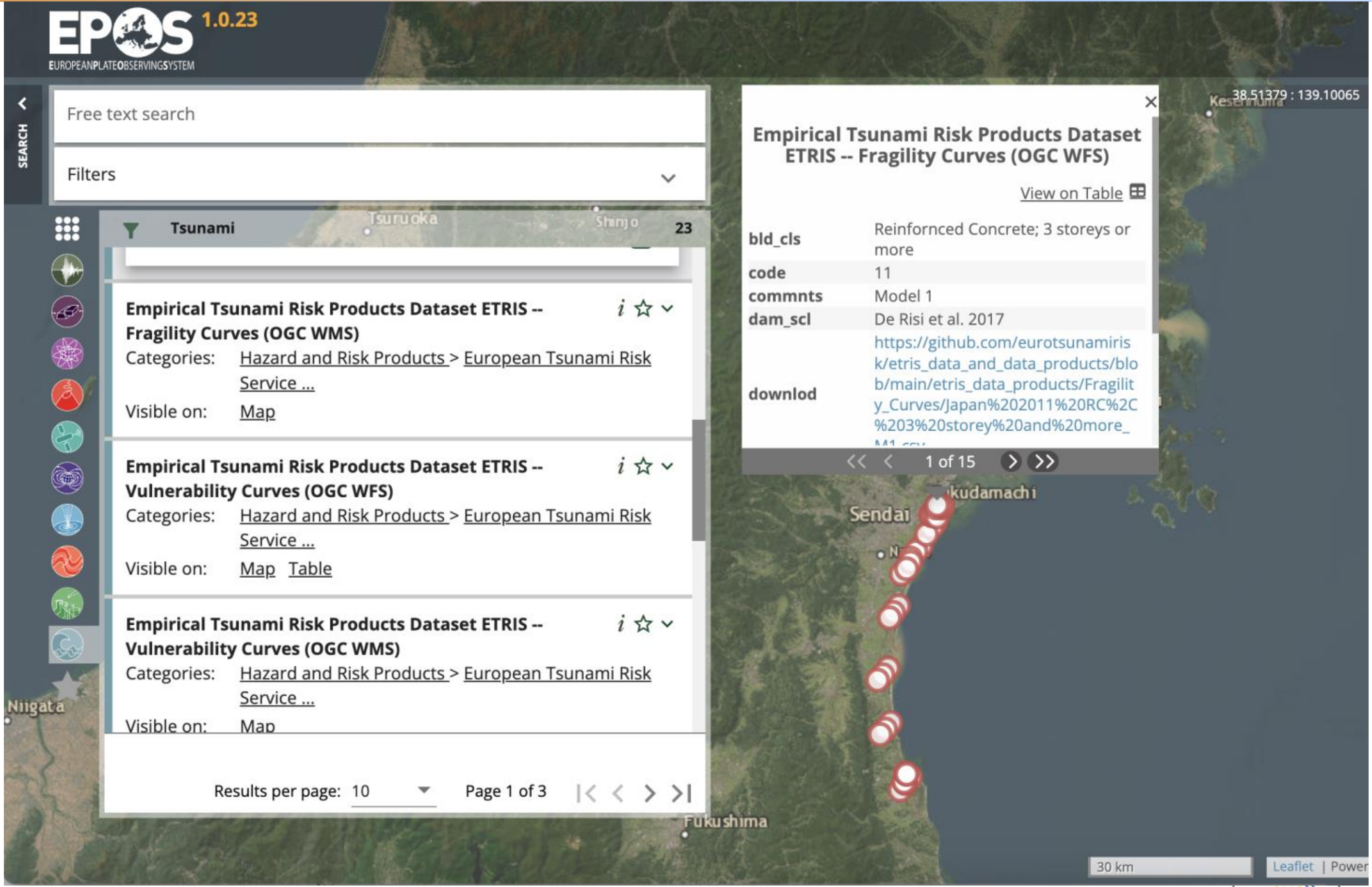
Equivalent Lognormal Statistics for Fragility Curve

- β_{UF} can be estimated as half of the (natural) logarithmic distance (along the *IM* axis) between the median intensities (i.e., 50% probability) of the fragility curves derived with 16% (denoted as IM^{84}) and 84% (IM^{16}) confidence levels, respectively; i.e., $\beta_{UF} = 0.50 \times \ln(IM^{84}/IM^{16})$. .



The Empirical Tsunami Risk Products – Fragility Layer

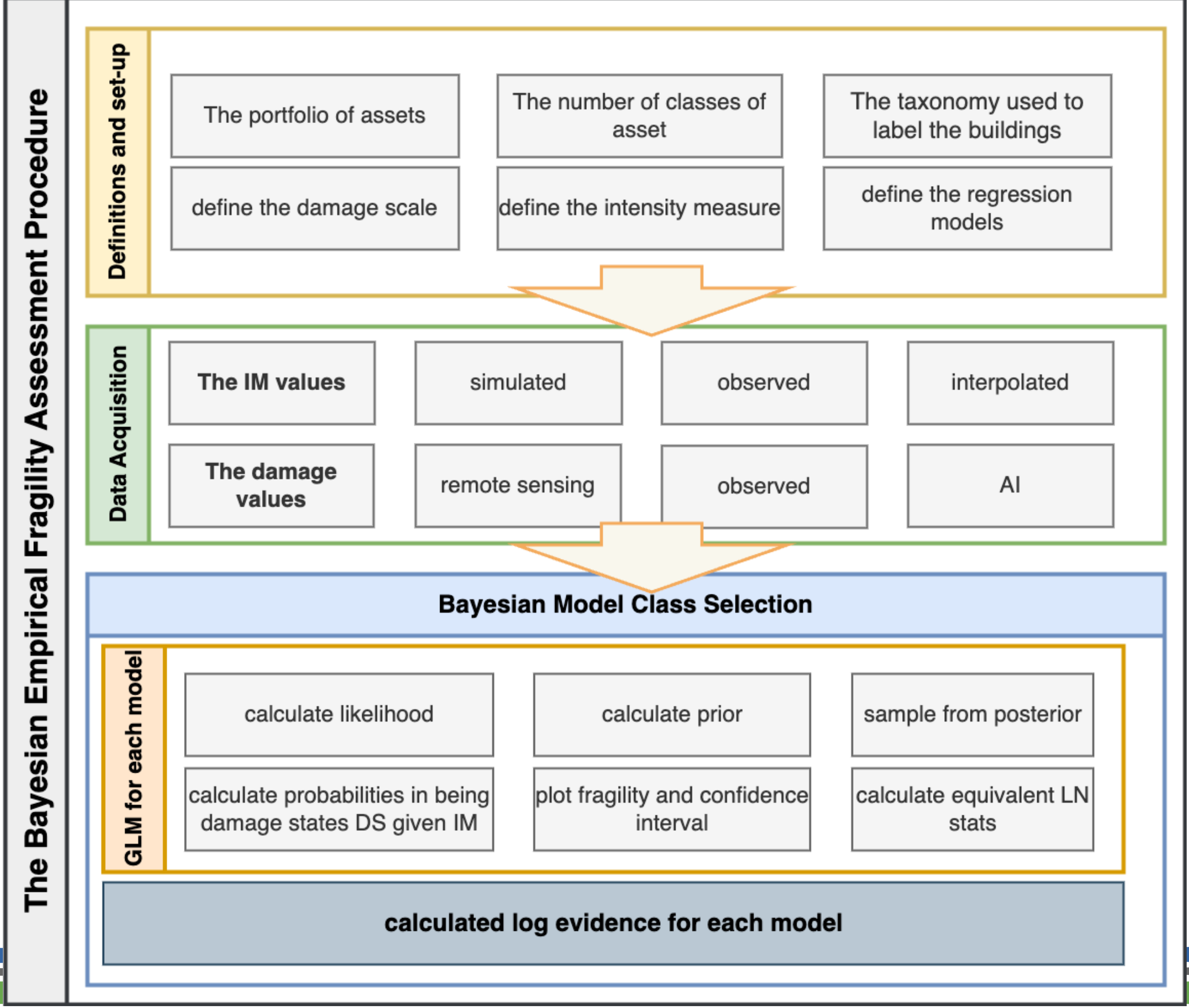
EPOS-ICS-S Portal



Geo-localization of Class Fragility Data

- Considering the definition of the class fragility curves, it is important to note that the geo-localization of such data is usually not accurate because these curves (by definition) represent a portfolio of spatially distributed assets.

- The geo-localization, if possible, is indicative of the geographic area where the surveyed assets are located.


The Bayesian Empirical Fragility Assessment Procedure
Definitions and set-up
The portfolio of assets
The number of classes of asset
The taxonomy used to label the buildings
define the damage scale
define the intensity measure
define the regression models
Data Acquisition
The IM values
simulated
observed
interpolated
The damage values
remote sensing
observed
AI
Bayesian Model Class Selection
GLM for each model
calculate likelihood
calculate prior
sample from posterior
calculate probabilities in being damage states DS given IM
plot fragility and confidence interval
calculate equivalent LN stats
calculated log evidence for each model

The procedure for obtaining the empirical fragility curves

- Establish the class or classes of asset for which the fragility curves are to be obtained.
- Establish a spatial extent for gathering the empirical data. This extent should be large enough to represent the class(es) of interest and the range of damage states/levels data gathered for this class.
- Very important: Establish the “base domain”. This essentially delineates the entire portfolio of assets being surveyed for damage.

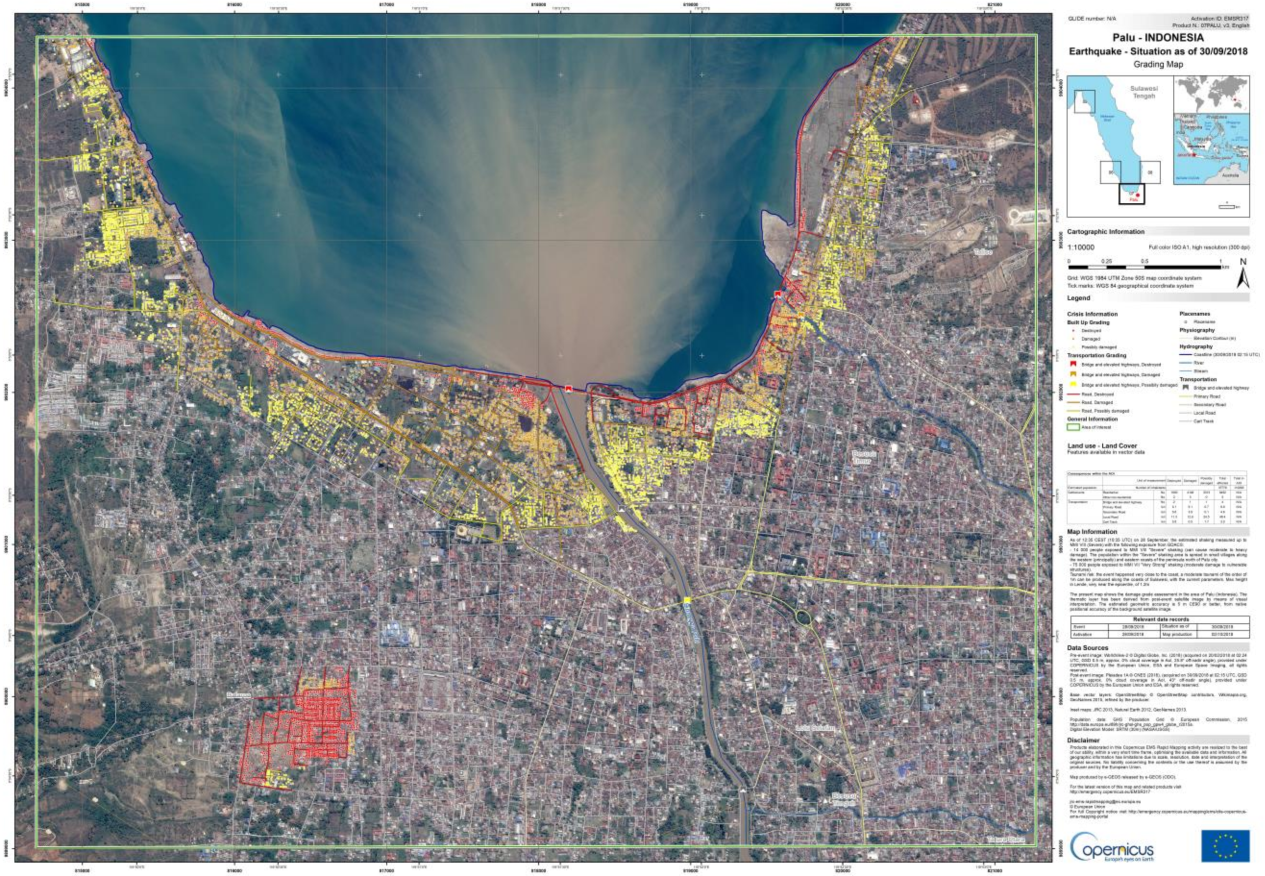
The procedure for obtaining the empirical fragility curves (continued)

- Obtain the empirical damage data:
  - By performing field surveys.
  - Through remote sensing data (e.g., EMS-Copernicus Damage Grading Maps).
  - Through machine-learning and AI.
  - By using existing damage data from literature.

- For example, damage data from Southeast Pacific Tsunami Event

Reese, Stefan, Brendon A. Bradley, Jochen Bind, Graeme Smart, William Power, and James Sturman. "Empirical building fragilities from observed damage in the 2009 South Pacific tsunami." *Earth-Science Reviews* 107, no. 1-2 (2011): 156-173.

Damage data from Copernicus Grading Map



Copernicus EMSR317
Palu Grading Map

The procedure for obtaining the empirical fragility curves (continued)

- Obtain the intensity values at points of interest:
  - By performing field surveys and measurements
  - By spatial averaging (smoothing) of the field surveys and measurements
  - Through numerical simulation

The different types of survey data

- **Aggregated**: where the IM information is reported as bins. In this case, the damage data will report the number of exposed asset in that IM bin that exceed a certain damage level (example, MLIT).

- **Point-wise**: In this case the IM values are reported at the position of each building (example Reese et al. 2011).

- The generalized regression routine presented here is applicable to both types of survey data.

The probability of being in a damage state DS given IM

- $P(D \geq D_j | IM)$ is the fragility function for damage level D_j.

$$P(DS_j | IM) = P\left[(D \geq D_j) \cdot (D < D_{j+1}) | IM \right]$$

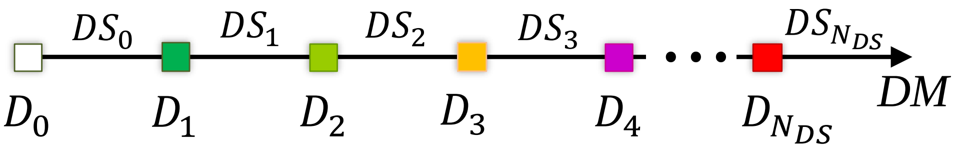
$$= \begin{cases} P(D \geq D_j | IM) - P(D \geq D_{j+1} | IM) & \text{for } 0 \leq j < N_{DS} \\ P(D \geq D_j | IM) & \text{for } j = N_{DS} \end{cases}$$

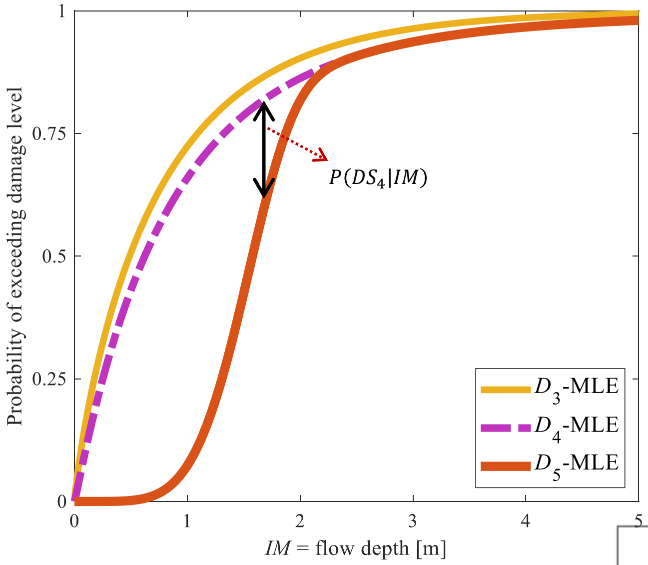
The representation of the fragility curve as P(DS|IM)

- The probability mass function definition is used for providing the probability of a discrete variable; e.g., being in a given damage state DS.

$$P\left(DS_j \middle| IM = im\right) = P\left(D > D_j \middle| IM = im\right) - P\left(D > D_{j+1} \middle| IM = im\right)$$

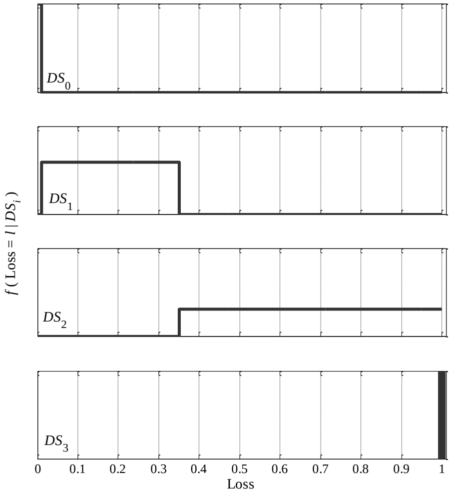
$$P\left(DS_{N_{DS}} \middle| IM = im\right) = P\left(D > D_{N_{DS}} \middle| IM = im\right)$$

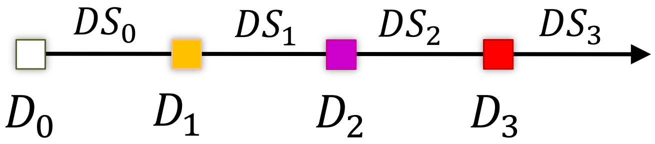




The Consequence Curve: Definition $G_{DV|DS}(dv \mid DS)$

- The consequence function can be formally defined as the probability distribution for the decision variable DV for (or conditioned on) a given damage state DS.





$$G(dv|DS_i) = P(DV > dv|DS_i)$$

$$f(DV = dv|DS = DS_i) = \left|\frac{dG(dv|DS_i)}{d\,dv}\right|$$

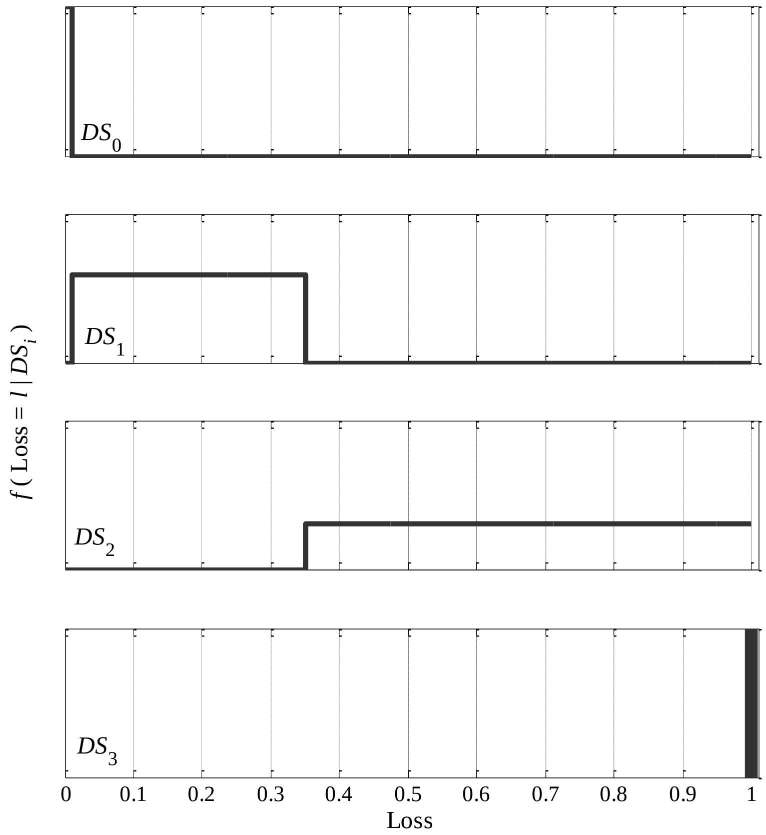
The consequence function (masonry buildings)

Damage_level_(D)	0	1	2	3	4	5
Damage_factor	0	0.01	0.1	0.35	0.75	1

Damage Level		Damage level description	
DS_0	None	no damage	
DS_1	Minor	Negligible to slight damage (no structural damage; slight non-structural damage)	Hair-line cracks in very few walls. Fall of small pieces of plaster only. Fall of loose stones from upper parts of buildings in very few cases.
DS_2	Moderate	Moderate damage (slight structural damage; moderate non-structural damage)	Cracks in many walls. Fall of fairly large pieces of plaster. Partial collapse of chimneys.
DS_3	Major	Substantial to heavy damage (moderate structural damage; heavy non-structural damage)	Large and extensive cracks in most walls. Roof tiles detach. Chimneys fracture at the roof line; failure of individual non-structural elements (partitions; gable walls).
DS_4	Severe	Very heavy damage (heavy structural damage; very heavy non-structural damage)	Serious failure of walls; partial structural failure of roofs and floors.
DS_5	Collapse	Destruction (very heavy structural damage)	Total or near total collapse.
		EMS 98 Damage Scale Masonry	

Damage Level		Damage level description
D_0	None	no damage
D_1	Repairable	Partial damage, repairable
D_2	Unrepairable	Partial damage, unrepairable
D_3	Complete	Complete structural collapse
		Sulawesi 2018 Paulik et al. 2019

Pasquale, G.D., Orsini, G. and Romeo, R.W., 2005. New developments in seismic risk assessment in Italy. *Bulletin of Earthquake Engineering*, *3*(1), pp.101-128.

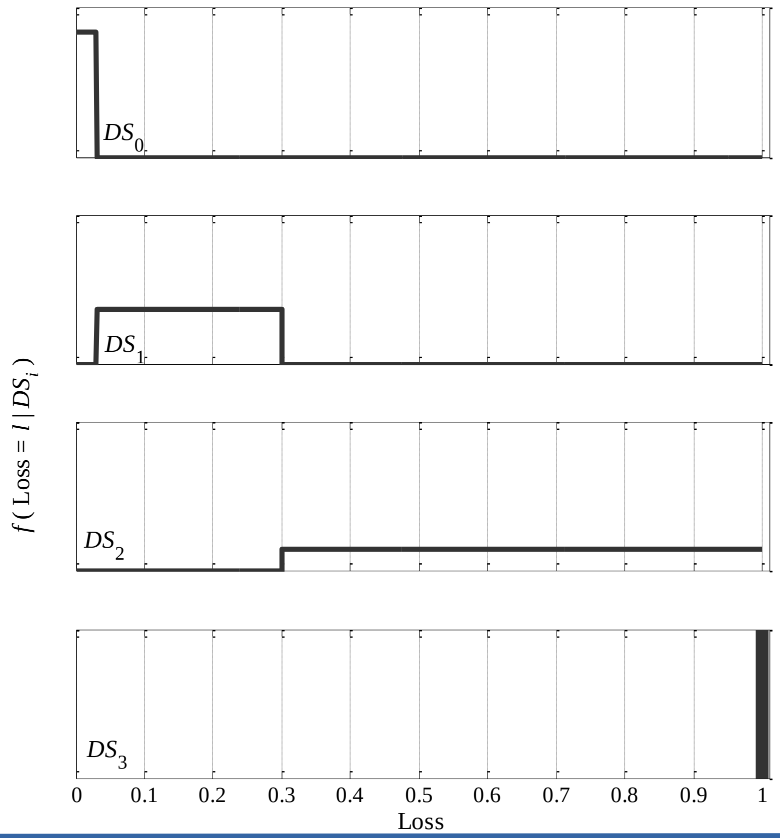


The consequence function (masonry buildings)

Damage Level		Damage level description
D_0	None	no damage
D_1	Repairable	Partial damage, repairable
D_2	Unrepairable	Partial damage, unrepairable
D_3	Complete	Complete structural collapse
		Sulawesi 2018 **Paulik et al. 2019**

The consequence function (timber buildings)

- Tsunami Damage ratios from Goda et al. (2021) based on MLIT damage scale: DS_0 (0-0.03), DS_1(0.03–0.1, 0.1–0.3), DS_2(0.3–0.5, 0.5–1.0), and DS_3(1.0), respectively for prevalently timber buildings.



Goda, K., De Risi, R., De Luca, F., Muhammad, A., Yasuda, T., & Mori, N. (2021). Multi-hazard earthquake-tsunami loss estimation of Kuroshio Town, Kochi Prefecture, Japan considering the Nankai-Tonankai megathrust rupture scenarios. *International Journal of Disaster Risk Reduction*, *54*, 102050.

Data Products for Tsunami Risk Assessment

- https://eurotsunamirisk.org/dataproducts/
- Some Examples of Consequence Functions in the Literature

Note that the consequence functions are not always formally defined as such, they can also be called cost model or the cost function.

Definition of the Vulnerability Curve

- The vulnerability curve can be defined as the probability of distribution for the decision variable DV given the intensity measure IM.

$$G_{DV|IM}(\mathrm{dv}|\mathrm{im}) = P(DV > dv|IM = im)$$

$$G_{DV|IM}(\mathrm{dv}|\mathrm{im}) = \sum_{j=0}^{N_{DS}} G_{DV|DS_j}(\mathrm{dv}|DS_j)P(DS_j|IM = im)$$

Vulnerability curves

$$G_{DV|IM}(\mathrm{dv}|\mathrm{im}) = \sum_{j=0}^{N_{DS}} G_{DV|DS_j}(\mathrm{dv}|DS_j)P(DS_j|IM = im)$$

$$P(DS_j|IM = im)$$

(Fragility Function)

DM ***(Damage Measure)***

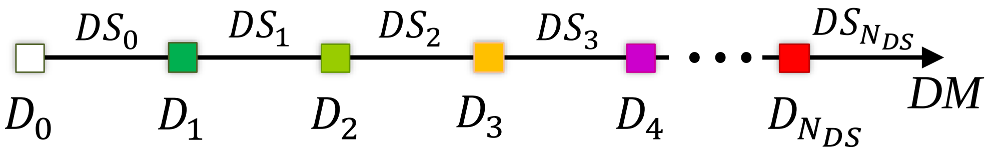
e.g., damage states

$$G_{DV|DS_j}(\mathrm{dv}|DS_j)$$

(Consequence Function)

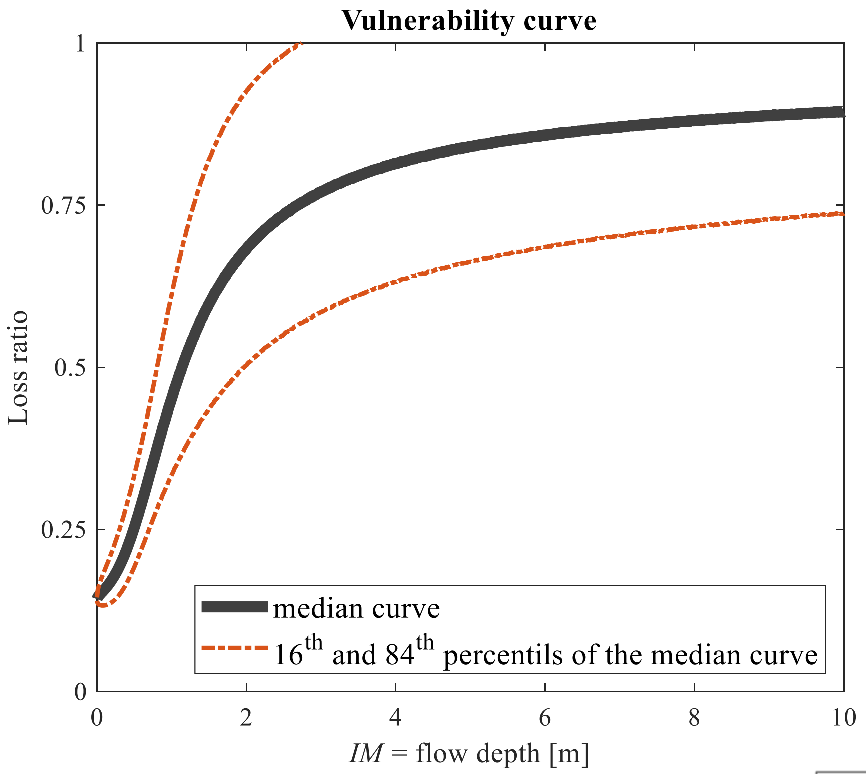
DV ***(Decision Variable)***

e.g., fatalities, loss

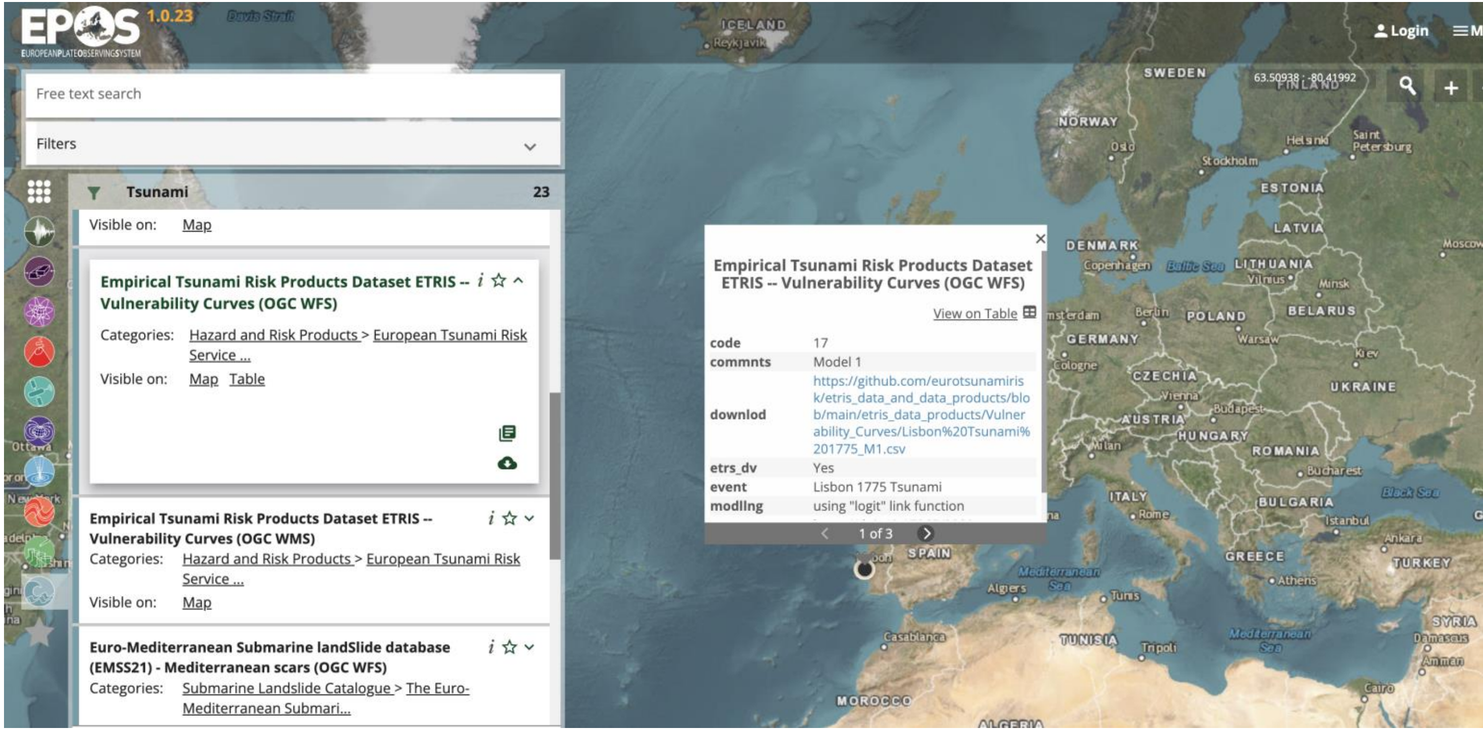


Definition of the Vulnerability Curve

- It is common to plot the vulnerability curve as the expected value (or median) and standard deviation of the decision variable DV (e.g., loss, fatalities) as a function of the intensity measure IM.



The Empirical Tsunami Risk Products – Vulnerability Layer



[EPOS-ICS-S Portal]

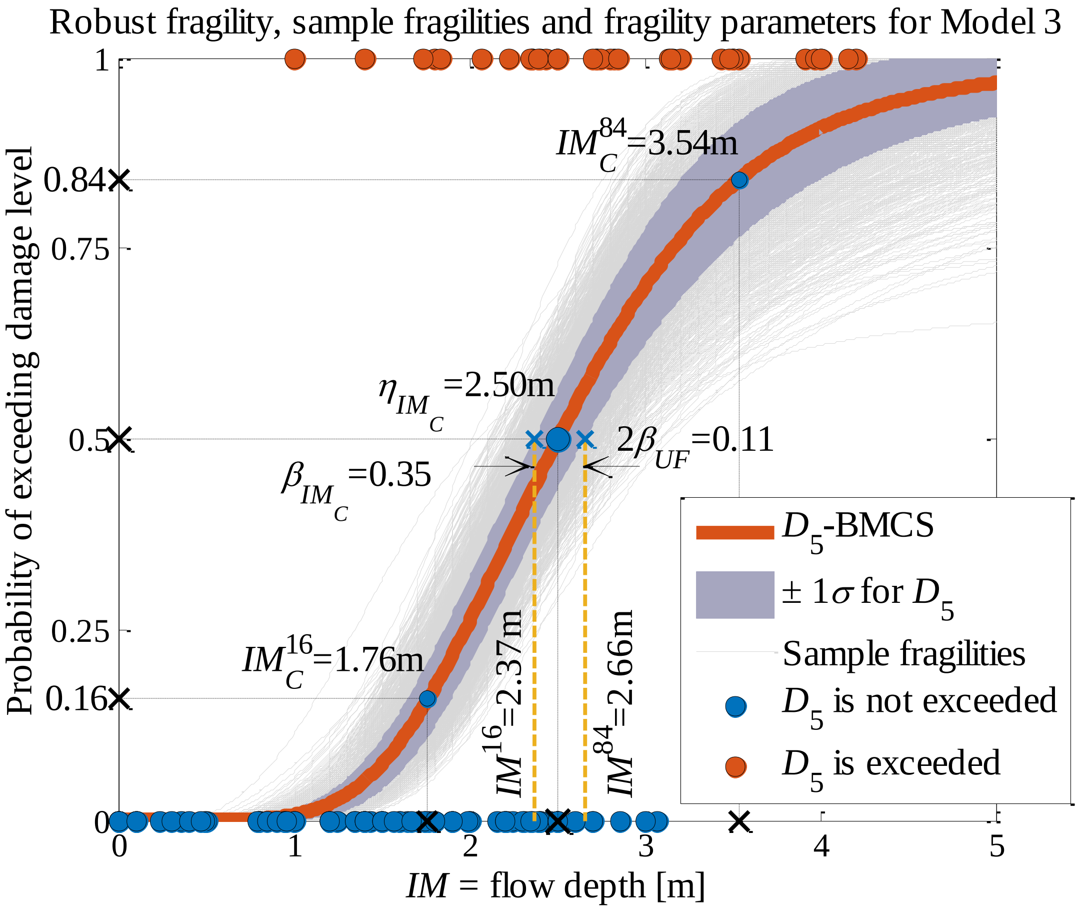
Using generalized regression for empirical fragility assessment

The Bernoulli Variable Y

- For damage level D_j, all exposed asset with an observed damage level $D < D_j$ will have a probability equal to zero, while those with $D \geq D_j$ will have an assigned probability equal to one. In other words, for asset *i* and damage state *j*, the Bernoulli variable Y_{ij} indicates whether asset *i* is in damage state *j*:

$$Y_{ij} = \begin{cases} 1 & \text{if asset } i \text{ exceeds } D_j & \text{with probability } P\left(D \geq D_j \mid IM_i\right) \\ 0 & \text{if asset } i \text{ does not exceed } D_j & \text{with probability } 1 - P\left(D < D_j \mid IM_i\right) \end{cases}$$

Visualisation: The Bernoulli Variable Y



Fragility Assessment for Hierarchical Damage Levels

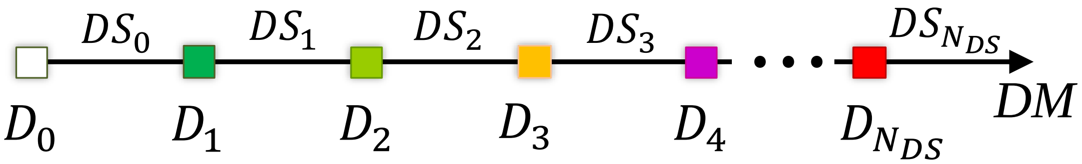
- According to this method, fragility assessment is going to be performed for all the damage states as an ensemble.

$$P(DS_j|IM_i) = P\left[(D<D_{j+1})\cdot(D\geq D_j)|IM_i\right]$$

$$= \left[1-P\left(D\geq D_{j+1}|D\geq D_j, IM_i\right)\right]\cdot P\left(D\geq D_j|IM_i\right)$$

Conditional Fragility: $\left[1-P\left(D\geq D_{j+1}|D\geq D_j, IM_i\right)\right]$

Recursive Formulation: $P\left(D\geq D_j|IM_i\right)$



Fragility Assessment using Generalized Regression

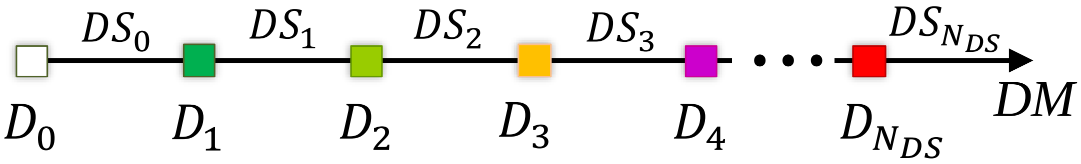
- Definition of viable fragility models:

$$l_{ij} = \alpha_{0,j} + \alpha_{1,j} \ln IM_i$$

$$\pi_{ij} = \pi_j(IM_i) = \begin{cases} \left(1+\exp(-l_{ij})\right)^{-1} & \text{logit} & \mathbb{M}_1 \\ \Phi\left(l_{ij}\right) & \text{probit} & \mathbb{M}_2 \\ 1-\exp\left(-\exp(l_{ij})\right) & \text{cloglog} & \mathbb{M}_3 \end{cases}$$

$$\pi_{ij} = P\left(D \geq D_{j+1} \middle| D \geq D_j, IM_i\right)$$

$$P\left(DS_j \middle| IM_i\right) = \left[1-\pi_{ij}\right] \cdot P\left(D \geq D_j \middle| IM_i\right)$$



Fragility Assessment using Generalized Regression

- The probabilities of being in different damage states can be calculated in a recursive way:

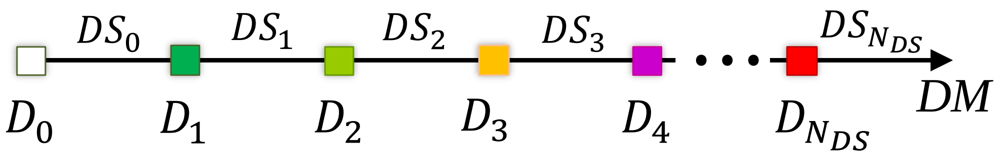
$$P\left(DS_j \middle| IM_i\right)=\begin{cases}\left(1-\pi_{ij}\right)\cdot\left[1-\sum_{k=0}^{j-1}P\left(DS_k \middle| IM_i\right)\right] & \text{for } j\geq 1\\ 1-\pi_{i0}\triangleq P\left(D<D_1 \middle| IM_i\right) & \text{for } j=0\end{cases}$$

$$P\left(DS_{N_{DS}} \middle| IM_i\right)=P\left(D\geq D_{N_{DS}} \middle| IM_i\right)=1-\sum_{j=0}^{N_{DS}-1}P\left(DS_j \middle| IM_i\right)$$

$$P\left(D\geq D_j \middle| IM_i\right)=P\left(DS_j \middle| IM_i\right)+P\left(D\geq D_{j+1} \middle| IM_i\right) \quad \text{for } 0\leq j<N_{DS}$$

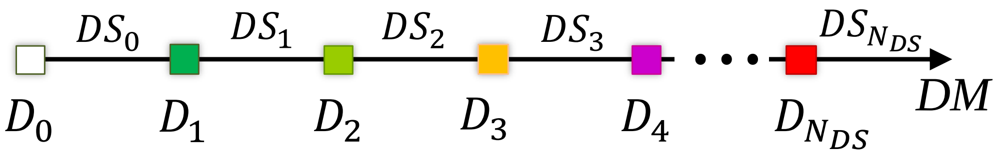
Data for the point-wise case

- Data is defined as couples of (IM_i, D_i) obtained at the position i of each asset at risk.
- Position *i* can be defined by latitude and longitude of the point.
- $n_{CL,j}$ is the number of assets from typology *CL* in damage state DS_j
- Where *i* is going to vary from 1 to the number of buildings $n_{CL,j}$
- D_i can have values equal to D_0 to D_{NDS}



What if data for all damage states are not available?

- Let ***index*** be the vector of values indicating damage levels for which observed data are available. The new damage scale formed as $\{DS_{index(1)}, DS_{index(2)}, \ldots, DS_{index(N)}\}$, where *N* is the length of vector ***index***, is also MECE.



Likelihood Calculation

- The likelihood is calculated as the probability of observing:

$$p(\mathbf{D} \mid \boldsymbol{\theta}_k, \mathbb{M}_k) = \prod_{j=0}^{N_{DS}} \prod_{i=1}^{n_{CL,j}} P\left(DS_j \middle| IM_i\right)$$

- The vector of fragility model parameters θ_k
- $\theta = \left[\alpha_{0,1}, \alpha_{1,1}, \alpha_{0,2}, \alpha_{1,2,\ldots,}\alpha_{0,N_{DS-1}}, \alpha_{1,N_{DS-1}}\right]$
- M_k is Model *k*, e.g., *k*=1:3

Likelihood Calculation for more than one class

- The likelihood is calculated as the probability of observing:

$$p(D|\theta_k, M_k) = \prod_{l=1}^{N_{CL}} \prod_{j=0}^{N_{DS}} \prod_{i=1}^{n_{CL_l,j}} P(DS_j\,|IM_i, CL_l)$$

  - M_k is Model *k*, e.g., *k*=1:3
  - N_{CL} is the number of building classes
  - $n_{CL_l,j}$ is the number of assets from typology CL_l in damage state DS_j
  - The vector of fragility model parameters θ_k
  - $\theta_k = \left[\alpha_{0,1}, \alpha_{1,1}, \alpha_{0,2}, \alpha_{1,2,\ldots,}\alpha_{0,N_{DS-1}}, \alpha_{1,N_{DS-1}}\right]$

The maximum likelihood estimation method

- A generalized linear regression model is used for the conditional fragility term $\pi_{ij} = P(D \geq D_{j+1} | D \geq D_j, IM)$ for the j^{th} damage state DS_j, $0 \leq j < N_{DS}$.

- Herein, we need to work with partial damage data so that all assets in DS_j (with an observed damage $D_j \leq D < D_{j+1}$) will be assigned a probability equal to zero, while those in higher damage states (with $D \geq D_{j+1}$) will be assigned a probability equal to one (i.e., to model the conditioning on $D \geq D_j$, the domain of possible damage levels is reduced to $D \geq D_j$).

Bayesian model class selection (BMCS)

- We use the Bayesian model class selection (BMCS) to identify the best link model to use in the generalized linear regression scheme.

- However, the procedure is general and can be applied to a more diverse pool of candidate fragility models.

- BMCS (or model comparison) is essentially Bayesian updating at the model class level to make comparisons among candidate model classes given the observed data

Bayesian Model Class Selection

- Given a set of $N_{\mathbb{M}}$ candidate model classes $\{\mathbb{M}_k, k = 1{:}\, N_{\mathbb{M}}\}$, and in the presence of the data **D**, the posterior probability of the k^{th} model class, denoted as $P(\mathbb{M}_k|\mathbf{D})$ can be written as follows:

$$P\left(\mathbb{M}_k \middle| \mathbf{D}\right) = \frac{p\left(\mathbf{D} \middle| \mathbb{M}_k\right) P\left(\mathbb{M}_k\right)}{\sum_{k=1}^{N_{\mathbb{M}}} p\left(\mathbf{D} \middle| \mathbb{M}_k\right) P\left(\mathbb{M}_k\right)}$$

Bayesian Model Class Selection

- Let us define the vector of model parameters $\boldsymbol{\theta}_k$ for model class $\mathbb{M}_k$ as $\boldsymbol{\theta}_k = \left[\left\{\alpha_{0,j}, \alpha_{1,j}\right\}_k, j = 0{:}\, N_{DS} - 1\right]$. We use the Bayes theorem to write the "*evidence*" $p(\mathbf{D}|\mathbb{M}_k)$ provided by data **D** for model $\mathbb{M}_k$ as follows:

$$p\left(\mathbf{D}\middle|\mathbb{M}_k\right) = \frac{p\left(\mathbf{D}\middle|\boldsymbol{\theta}_k, \mathbb{M}_k\right) p\left(\boldsymbol{\theta}_k\middle|\mathbb{M}_k\right)}{p\left(\boldsymbol{\theta}_k\middle|\mathbf{D}, \mathbb{M}_k\right)}$$

The (log) evidence

- that logarithm of the evidence (called *log-evidence*) $\ln[p(\mathbf{D}|\mathbb{M}_k)]$ can be written as:

$$\ln\left[p\left(\mathbf{D}\middle|\mathbb{M}_k\right)\right] = \underbrace{\int_{\Omega_{\boldsymbol{\theta}_k}} \ln\left[p\left(\mathbf{D}\middle|\boldsymbol{\theta}_k,\mathbb{M}_k\right)\right] p\left(\boldsymbol{\theta}_k\middle|\mathbf{D},\mathbb{M}_k\right) d\boldsymbol{\theta}_k}_{Term\ 1} - \underbrace{\int_{\Omega_{\boldsymbol{\theta}_k}} \ln\left[\frac{p\left(\boldsymbol{\theta}_k\middle|\mathbf{D},\mathbb{M}_k\right)}{p\left(\boldsymbol{\theta}_k\middle|\mathbb{M}_k\right)}\right] p\left(\boldsymbol{\theta}_k\middle|\mathbf{D},\mathbb{M}_k\right) d\boldsymbol{\theta}_k}_{Term\ 2}$$

- "Term 1" denotes the posterior mean of the log-likelihood, which is a measure of the average data fit to model $\mathbb{M}_k$."Term 2" is the relative entropy between the prior $p(\boldsymbol{\theta}_k|\mathbb{M}_k)$ and the posterior $p(\boldsymbol{\theta}_k|\mathbf{D},\mathbb{M}_k)$ of $\boldsymbol{\theta}_k$ given model $\mathbb{M}_k$, which is a measure of the distance between the two PDFs.

The Posterior Distribution for Fragility Model Parameters

- The posterior distribution $p(\boldsymbol{\theta}_k|\mathbf{D}, \mathbb{M}_k)$ can be found based on Bayesian inference:

$$\underbrace{p\left(\boldsymbol{\theta}_k \middle| \mathbf{D}, \mathrm{M}_k\right)}_{\text{posterior}} = \frac{p\left(\mathbf{D}\middle|\boldsymbol{\theta}_k, \mathrm{M}_k\right) p\left(\boldsymbol{\theta}_k \mid \mathrm{M}_k\right)}{\int_{\Omega_{\boldsymbol{\theta}_k}} p\left(\mathbf{D}\middle|\boldsymbol{\theta}_k, \mathrm{M}_k\right) p\left(\boldsymbol{\theta}_k \mid \mathrm{M}_k\right) d\boldsymbol{\theta}_k} = C^{-1} \underbrace{p\left(\mathbf{D}\middle|\boldsymbol{\theta}_k, \mathrm{M}_k\right)}_{\text{likelihood}} \underbrace{p\left(\boldsymbol{\theta}_k \mid \mathrm{M}_k\right)}_{\text{prior}}$$

where C^{-1} is a normalizing constant.

Markov Chain Monte Carlo Simulation Routine

- The prior distribution, $p(\boldsymbol{\theta}_k|\mathbb{M}_k)$, can be estimated as the product of marginal normal PDFs for each model parameter, i.e., a multivariate normal distribution with zero correlation between the pairs of model parameters $\boldsymbol{\theta}_k$.

- To sample from the posterior distribution $p(\boldsymbol{\theta}_k|\mathbf{D}, \mathbb{M}_k)$, an *adaptive* MCMC simulation routine is employed.

Markov Chain Monte Carlo Simulation Routine (continued)

- MCMC is particularly useful for drawing samples from the target posterior, while it is known up to a scaling constant C^{-1}.

- Thus, we only need un-normalized PDFs to feed the MCMC procedure. The MCMC routine herein employs the Metropolis-Hastings (MH) algorithm to generate samples from the target joint posterior PDF.

Robust Fragility Assessment

- Robust Fragility (RF) is defined as the expected value for a prescribed fragility model considering the joint probability distribution for the fragility model parameters $\boldsymbol{\theta}_k$. The RF herein can be expressed as:

$$P\left(D \geq D_j \middle| IM, \mathbf{D}, \mathbb{M}_k\right) = \int_{\Omega_{\boldsymbol{\theta}_k}} P\left(D \geq D_j \middle| IM, \boldsymbol{\theta}_k\right) p\left(\boldsymbol{\theta}_k \middle| \mathbf{D}, \mathbb{M}_k\right) \mathrm{d}\boldsymbol{\theta}_k = \mathbb{E}_{\boldsymbol{\theta}_k|\mathbf{D},\mathbb{M}_k}\left[P\left(D \geq D_j \middle| IM, \boldsymbol{\theta}_k\right)\right]$$

$$\sigma^2_{\boldsymbol{\theta}_k|\mathbf{D},\mathbb{M}_k}\left[P\left(D \geq D_j \middle| IM, \boldsymbol{\theta}_k\right)\right] = \underbrace{\mathbb{E}_{\boldsymbol{\theta}_k|\mathbf{D},\mathbb{M}_k}\left[P\left(D \geq D_j \middle| IM, \boldsymbol{\theta}_k\right)^2\right]}_{\cong \frac{1}{N_d}\sum_{i=1}^{N_d} P\left(D \geq D_j | IM, \boldsymbol{\theta}_{k,l}\right)^2} - \underbrace{\left(\mathbb{E}_{\boldsymbol{\theta}_k|\mathbf{D},\mathbb{M}_k}\left[P\left(D \geq D_j \middle| IM, \boldsymbol{\theta}_k\right)\right]\right)^2}_{= P\left(D \geq D_j | IM, \mathbf{D}, \mathbb{M}_k\right)^2 \ (\text{Eq.16})}$$

Using Monte Carlo Simulation for Fragility Assessment

- The RF integral can be solved numerically by employing Monte Carlo simulation with N_d generated samples from the vector $\boldsymbol{\theta}_k$ as follows:

$$P\left(D \geq D_j \middle| IM, \mathbf{D}, \mathbb{M}_k\right) \cong \frac{1}{N_d} \sum_{l=1}^{N_d} P\left(D \geq D_j \middle| IM, \boldsymbol{\theta}_{k,l}\right)$$

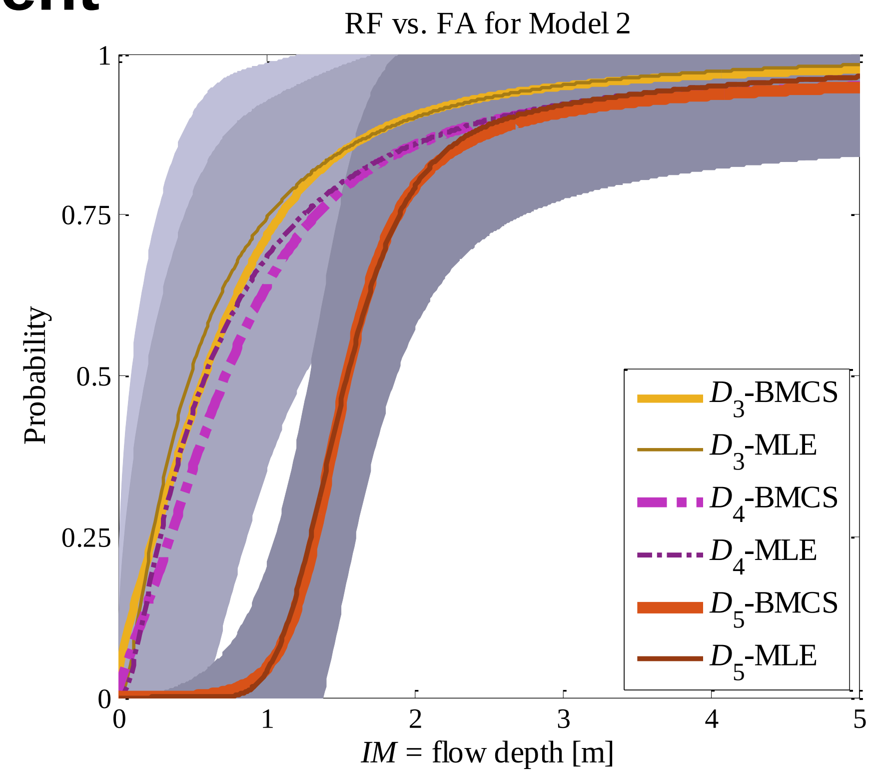
Using Monte Carlo Simulation for Fragility Assessment

- The integral equation for standard deviation of the fragility can be solved numerically by employing Monte Carlo simulation with N_d generated samples from the vector $\boldsymbol{\theta}_k$ as follows:

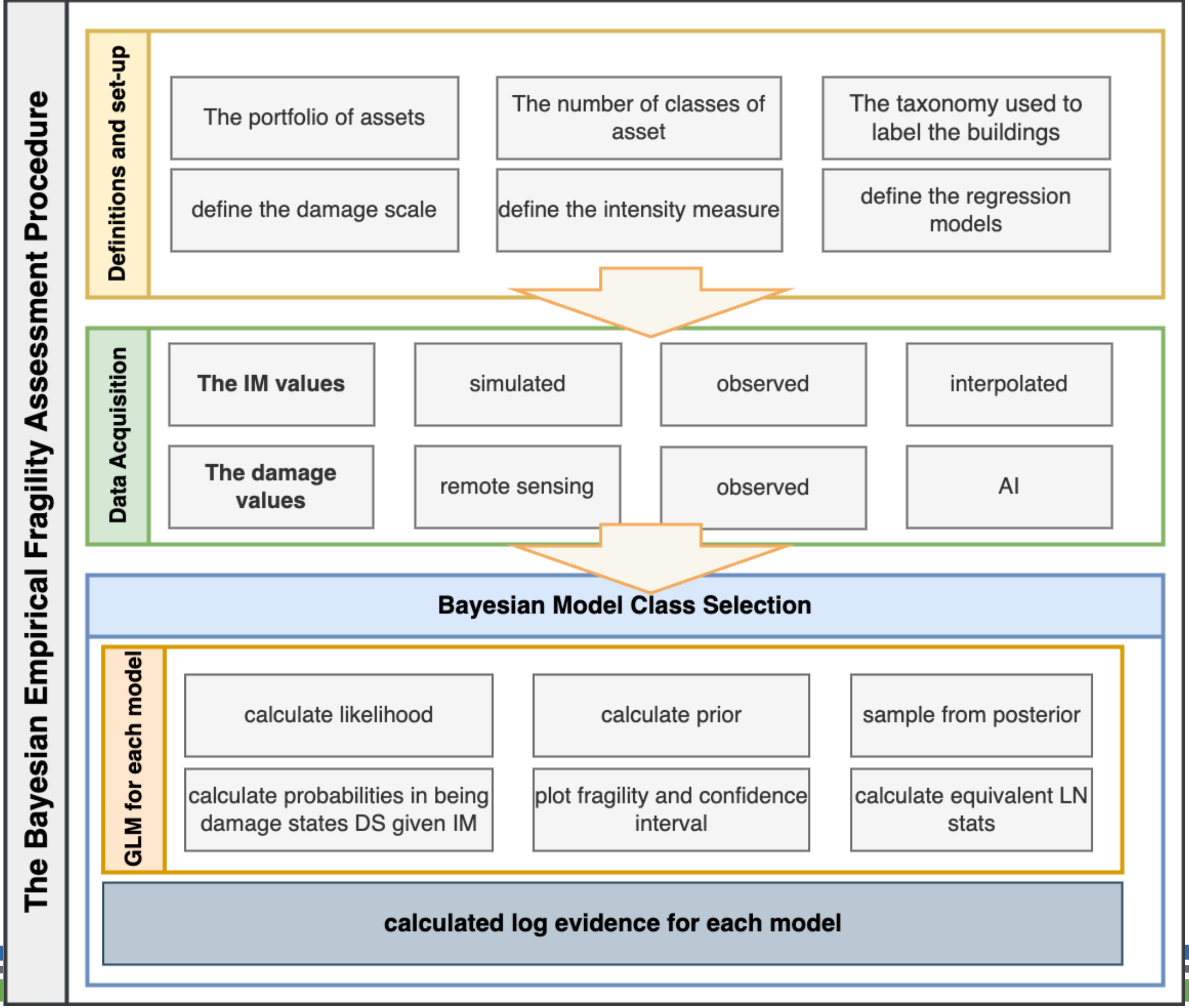
$$\sigma^2_{\boldsymbol{\theta}_k|\mathbf{D},\mathbb{M}_k}\left[P\left(D \geq D_j \middle| IM,\boldsymbol{\theta}_k\right)\right] \cong \frac{1}{N_d}\sum_{i=1}^{N_d} P\left(D \geq D_j \middle| IM,\boldsymbol{\theta}_{k,l}\right)^2 - P\left(D \geq D_j \middle| IM,\mathbf{D},\mathbb{M}_k\right)^2$$

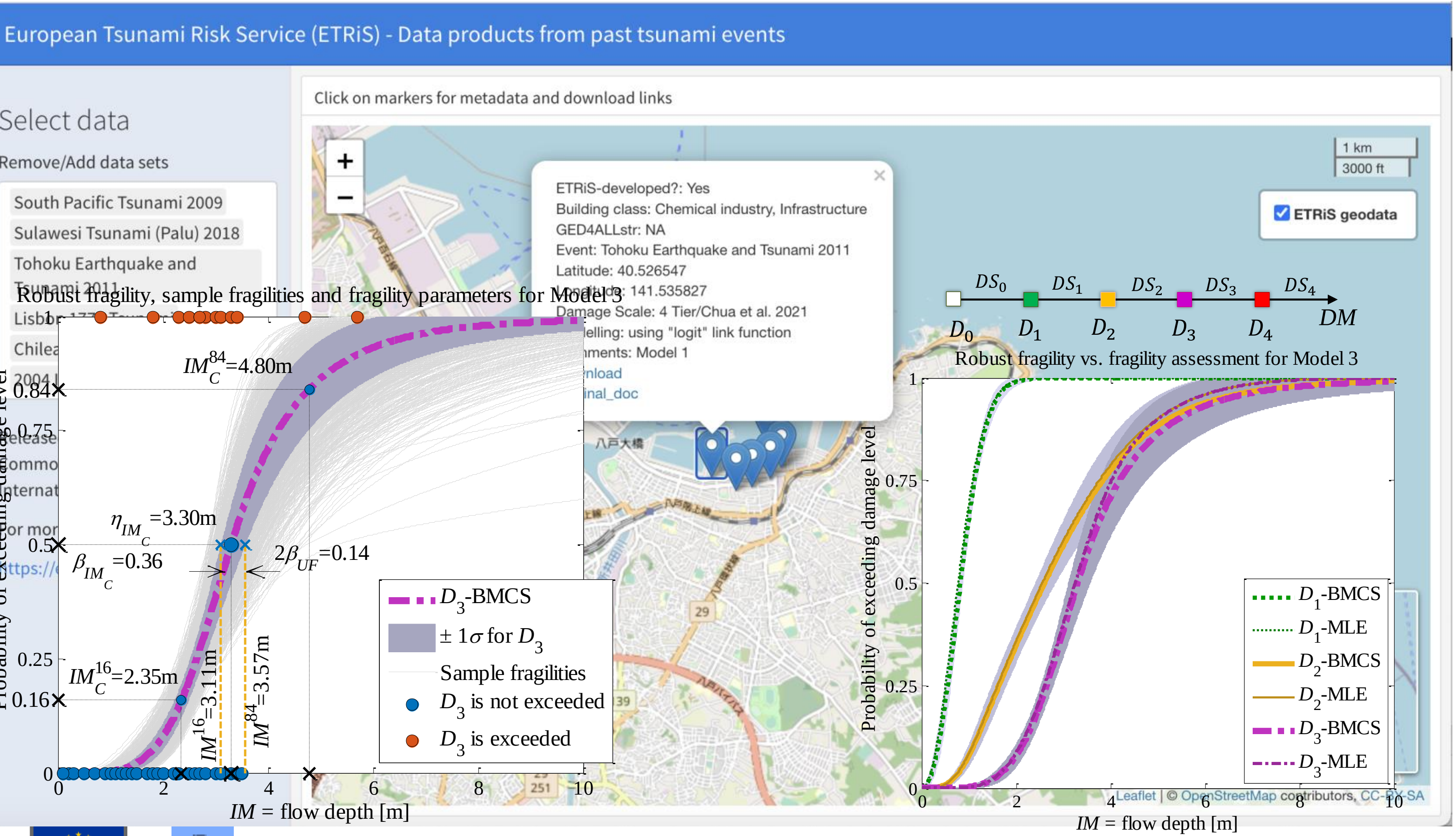
Robust Fragility Assessment

- RF is defined as the expected value for a prescribed fragility model considering the joint probability distribution for the fragility model parameters $\boldsymbol{\theta}_k$. The RF herein can be expressed as:



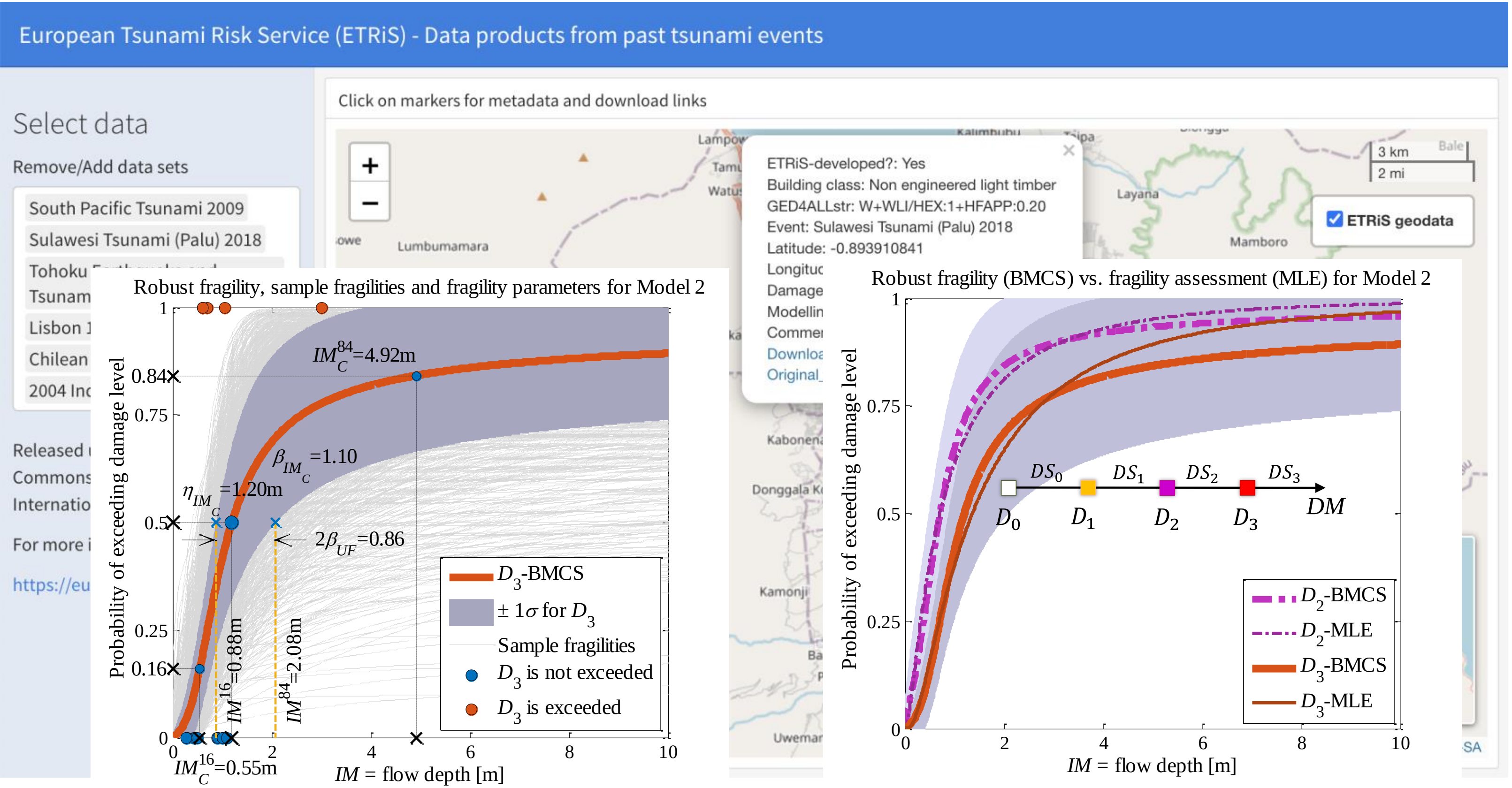
Recalling the Flowchart:





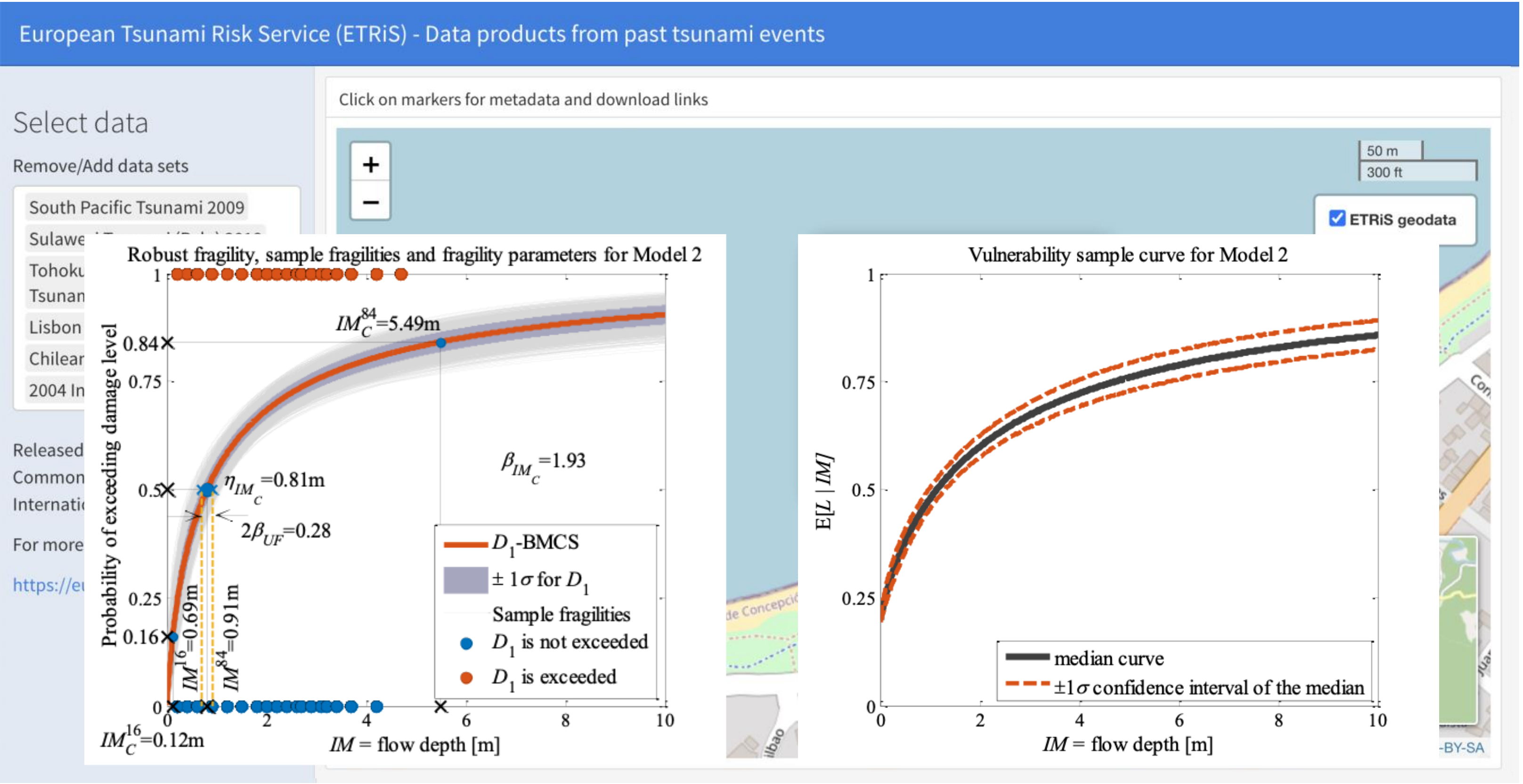
Fragility curves for Chemical Industry Infrastructure Damaged due to 2011 Great East Japan earthquake and tsunami

Chua, C. T., Switzer, A. D., Suppasri, A., Li, L., Pakoksung, K., Lallemant, D., ... & Winspear, N. (2021). Tsunami damage to ports: cataloguing damage to create fragility functions from the 2011 Tohoku event. *Natural Hazards and Earth System Sciences*, *21*(6), 1887-1908.



Fragility curves for non-engineered light timber damaged due to 2018 Sulawesi-Palu tsunami

Paulik, R., Gusman, A., Williams, J.H., Pratama, G.M., Lin, S.L., Prawirabhakti, A., Sulendra, K., Zachari, M.Y., Fortuna, Z.E.D., Layuk, N.B.P. and Suwarni, N.W.I., 2019. Tsunami hazard and built environment damage observations from Palu City after the September 28 2018 Sulawesi earthquake and tsunami. *Pure and Applied Geophysics*, *176*, pp.3305-3321.



Vulnerability curve based on data for the Chilean Tsunami 2010.

Mas, E., Koshimura, S., Suppasri, A., Matsuoka, M., Matsuyama, M., Yoshii, T., Jimenez, C., Yamazaki, F. and Imamura, F., 2012. Developing Tsunami fragility curves using remote sensing and survey data of the 2010 Chilean Tsunami in Dichato. *Natural Hazards and Earth System Sciences*, *12*(8), pp.2689-2697.

Useful references and documentation

- Jalayer, Fatemeh, Hossein Ebrahimian, Konstantinos Trevlopoulos, and Brendon Bradley. "Empirical tsunami fragility modelling for hierarchical damage levels." *Natural Hazards and Earth System Sciences* 23, no. 2 (2023): 909-931.
- Behrens, Jörn, Finn Løvholt, Fatemeh Jalayer, Stefano Lorito, Mario A. Salgado-Gálvez, Mathilde Sørensen, Stephane Abadie et al. "Probabilistic tsunami hazard and risk analysis: A review of research gaps." *Frontiers in Earth Science* 9 (2021): 628772.
- The European Tsunami Risk Service: https://eurotsunamirisk.org/
- EPOS ICS-C Portal: https://www.ics-c.epos-eu.org/

Some useful ETRiS Repositories

- Repository of fragility functions:

https://github.com/eurotsunamirisk/etris_data_and_data_products/tree/main/etris_data_products/Fragility_Curves

- Repository of Vulnerability Functions

https://github.com/eurotsunamirisk/etris_data_and_data_products/tree/main/etris_data_products/Vulnerability_Curves

- ComputeFrag Software

https://eurotsunamirisk.org/tsunamirisktoolkit/

- Map Viewer

https://eurotsunamirisk.org/maps/

- Jupyter Notebooks for Fragility Visualisation

https://github.com/eurotsunamirisk/VisualizeFragility

Geo-INQUIRE is a joint effort of 51 institutions

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