ETRiS - Geo-INQUIRE online training course , Second Day: November 7, 2023
Empirical fragility and vulnerability curves for risk analysis (VA2-35-1)

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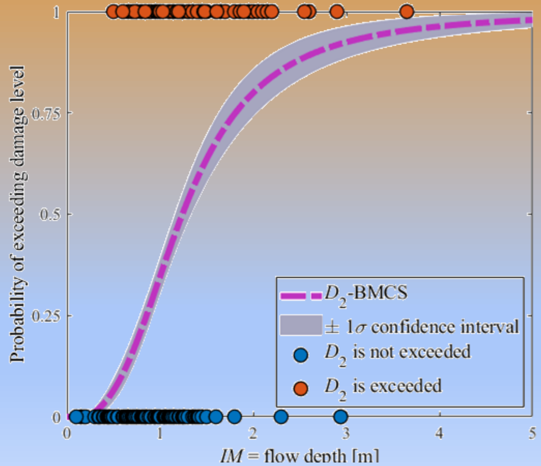

University College London
INSTITUTE FOR RISK AND DISASTER REDUCTION (IRDR)

European Tsunami Risk Service (ETRiS)
Candidate Thematic Core Service (cTCS-Tsu)

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We have seen in the First Day of the Training Course:

- The forward probabilistic framework
- Damage scales
- Definition of fragility function
- Empirical fragility assessment using GLM
- Bayesian model class selection
- The definition of vulnerability function

https://eurotsunamirisk.org/tsunamirisktoolkit/

computeFrag

Will be added to the ETRiS

If you are using computeFrag, you should cite this paper:

Jalayer, F., Ebrahimian, H., Trevlopoulos, K. and Bradley, B., 2023. Empirical tsunami fragility modelling for hierarchical damage levels. *Natural Hazards and Earth System Sciences*, *23*(2), pp.909-931. https://doi.org/10.5194/nhess-23-909-2023

ComputeFrag: an integrated workflow based on Bayesian model class selection for empirical hierarchical fragility modeling for a class of assets

The fragility model parameters are inferred using Maximum Likelihood Estimate (MLE) method or by using the Bayesian inference (BMCS) considering **Hierarchical fragility modelling**.

The fragility models are described using generalized linear model (GLM) with three different link functions: **logit, probit** and **cloglog**.

Selecting the simplest model that fits the data best amongst the suite of three candidate fragility models with alternative link functions described before.

Step 1

Input

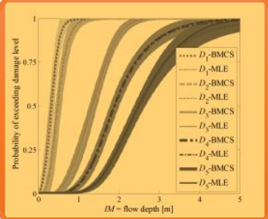
Intensity Measure (IM)

Damage Levels

Step 02

Fragility model parameter estimation

Step 03



Step 04

Bayesian Model Class Selection

The input is the vector of tsunami intensity measures (*IM*) and the corresponding observed damage levels (*D*) for the inspected buildings and infrastructures.

Fragility and confidence interval derivation by BMCS-based hierarchical fragility modelling

Fragility assessment by MLE-based hierarchical fragility modelling

https://eurotsunamirisk.org/tsunamirisktoolkit/

The architecture of computeFrag

https://github.com/eurotsunamirisk/computeFrag/tree/main/Code_ver2

Step 1: Input

The file **.csv** containing two columns, the first for the intensity measures and the second for the damage levels.

MLE-based hierarchical fragility modeling

Step 2: Estimating fragility model parameters

Using generalized linear regression (glm) based on Maximum Likelihood Estimation method (*MLE method*) considering **hierarchical fragility modeling**.

`calculate_glm.m`

Step 3: Fragility Assessment

Fragility assessment by MLE-based hierarchical fragility modeling

`calculate_fragility.m`

BMCS-based hierarchical fragility modeling

Step 2: Estimating fragility model parameters

Employing the Bayesian inference for model updating using adaptive MCMC to obtain the joint distribution of the model parameters (*BMCS method*) using **hierarchical fragility modeling**.

`posteriorFragilityFunction.m`

Step 3: Fragility Assessment

Robust Fragility assessment by BMCS-based hierarchical fragility modeling

`RobustFragility_DS.m`

Step 4: Bayesian Model Class Selection

Bayesian model class selection (BMCS) and parameter inference using adaptive MCMC

`calculate_loge.m`

MLE-Basic fragility modeling

Step 2: Estimating fragility model parameters

Using generalized linear regression (glm) based on maximum likelihood estimation (*MLE-Basic method*) and **fitting the fragility model to one damage state at a time**.

`calculate_glm_basic.m`

Step 3: Fragility Assessment

Fragility assessment by MLE-Basic method

`calculate_fragility_basic.m`

Input & Output Concept of computeFrage

Observed Tsunami intensity measures (*IM*)

Observed damage levels (*D*)

Version 1

- ➢ MATLAB version
- ➢ Standalone docker application that calls the MatLab engine

https://eurotsunamirisk.org/tsunamirisktoolki

Version 2

- ➢ MATLAB version

Version 3

Released soon

- Improvements to Version 2

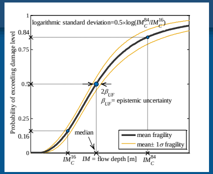
- Adding Figures to the output

Providing a docker application with Jupyter notebooks

Output version 1

- BMCS-based hierarchical fragility model parameters.

- Robust fragility assessment and its confidence interval.

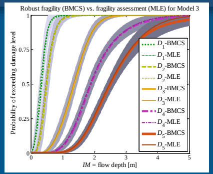


Output version 2

- MLE-based hierarchical fragility model parameters and fragility assessment.

- BMCS-based hierarchical fragility model parameters, Robust fragility assessment, and Bayesian model class selection.

- MLE-Basic fragility model parameters and fragility assessment.



Geo-INQUIRE

The probability of being in a damage state DS given IM

- $P(D \geq D_j|IM)$ is the fragility function for damage level D_j.

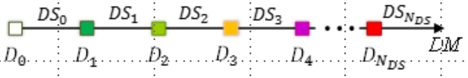
$$P(DS_j|IM) = P\left[(D \geq D_j)\cdot(D < D_{j+1})|IM\right]$$
$$= \begin{cases} P(D \geq D_j|IM) - P(D \geq D_{j+1}|IM) & \text{for } 0 \leq j < N_{DS} \\ P(D \geq D_j|IM) & \text{for } j = N_{DS} \end{cases}$$

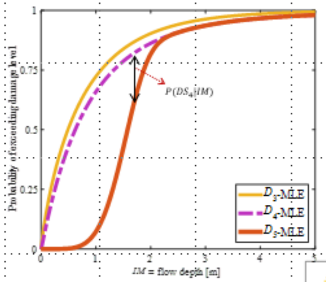
Geo-INQUIRE

The representation of the fragility curve as P(DS|IM)

- The probability mass function definition is used for providing the probability of a discrete variable; e.g., being in a given damage state DS.

$$P(DS_j|IM = im) =$$
$$P(D > D_j|IM = im) - P(D > D_{j+1}|IM = im)$$
$$P(DS_{N_{DS}}|IM = im) = P(D > D_{N_{DS}}|IM = im)$$





Damage State ← DS_0 DS_1 DS_2 DS_3 … $DS_{N_{DS}}$

Damage level ← D_0 D_1 D_2 D_3 D_4 … $D_{N_{DS}}$ DM

$$DS_j \equiv (D \ge D_j)\cdot(D < D_{j+1})$$

$$P(DS_j|IM) = P\left[(D \ge D_j)\cdot(D < D_{j+1})|IM\right]$$

$$P(DS_j|IM)$$

Basic Fragility Modeling

$$\pi_{ij} = P(D \ge D_j|IM_i)$$

$$= \begin{cases} P(D \ge D_j|IM) - P(D \ge D_{j+1}|IM) & \text{for } 0 \le j < N_{DS} \\ P(D \ge D_j|IM) & \text{for } j = N_{DS} \end{cases}$$

Viable Fragility Models

$$\pi_{ij} = \pi_j(IM_i) = \begin{cases} \left(1+\exp(-\alpha_{0,j} - \alpha_{1,j}\ln IM_i)\right)^{-1} & \text{logit} \rightarrow \mathbb{M}_1 \\ \Phi\left(\alpha_{0,j} + \alpha_{1,j}\ln IM_i\right) & \text{probit} \rightarrow \mathbb{M}_2 \\ 1-\exp\left(-\exp(\alpha_{0,j} + \alpha_{1,j}\ln IM_i)\right) & \text{cloglog} \rightarrow \mathbb{M}_3 \end{cases}$$

Hierarchical Fragility Modelling

$$\pi_{ij} = P\left(D \ge D_{j+1} \middle| D \ge D_j, IM_i\right)$$

$$= \underbrace{\left[1-P\left(D \ge D_{j+1}|D \ge D_j, IM_i\right)\right]}_{1-\pi_{ij}} \cdot \underbrace{\left[1-P\left(D < D_j|IM_i\right)\right]}_{\text{Recursive Formulation}}$$

Jalayer, F., Ebrahimian, H., Trevlopoulos, K. and Bradley, B., 2023. Empirical tsunami fragility modelling for hierarchical damage levels. *Natural Hazards and Earth System Sciences*, *23*(2), pp.909-931.

Hierarchical vs. Basic Fragility Modelling

DS_2 DS_3 DS_4 DS_5

D_2 D_3 D_4 D_5 DM

$P\left(DS_j|IM\right)=P\left[\left(D\geq D_j\right)\cdot\left(D<D_{j+1}\right)|IM\right]$

$P\left(DS_j|IM\right)$

Basic Fragility Modeling

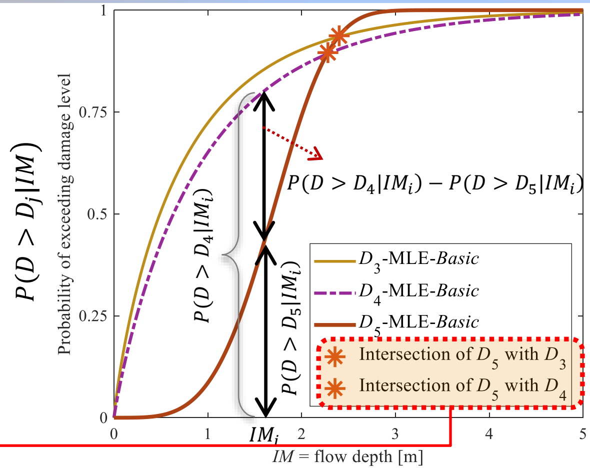
$\pi_{ij} = P\left(D \geq D_j|IM_i\right)$

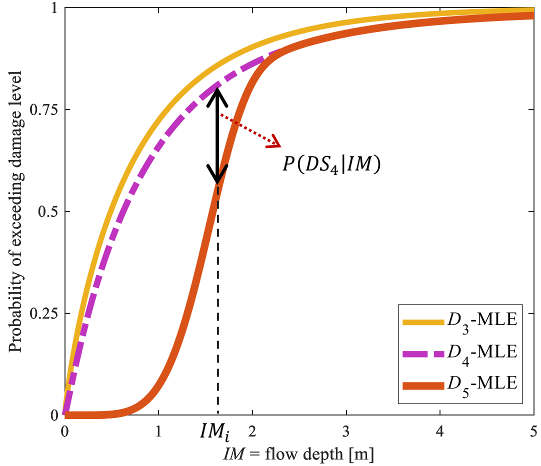
The ill condition that $P\left(DS_j|IM\right) < 0$

With this condition, we ensure that the fragility curve of a lower damage level will not fall below the fragility curve of the subsequent damage threshold.

Hierarchical Fragility Modelling

$\pi_{ij} = P\left(D \geq D_{j+1} \,\middle|\, \boxed{D \geq D_j}, IM_i\right)$





Building class 2 (Timber residential) of South Pacific 2009 Tsunami

Using MATLAB ToolBox

Hierarchical Fragility Modelling using MLE – Parameter Estimation

$$\pi_{ij} = P\left(D \geq D_{j+1} \middle| D \geq D_j, IM_i\right)$$

`calculate_glm.m`

All buildings in with observed damage $D_j \leq D < D_{j+1}$ will be assigned a probability equal to zero, while those with $D \geq D_{j+1}$ will be assigned a probability equal to one.

The vector of fragility model parameters is:

$$\mathbf{\theta} = \left[\{\alpha_{0,j}, \alpha_{1,j}\}, j = 0{:}\, N_{DS} - 1\right]$$

`theta_prior_modelk`
`k=1,2,3`

Basic Fragility Modelling using MLE (MLE-*Basic*) – Parameter Estimation

$$\pi_{ij} = P(D \geq D_j | IM_i)$$

`calculate_glm_basic.m`

All buildings in with observed damage $D < D_j$ will be assigned a probability equal to zero, while those with $D \geq D_j$ will be assigned a probability equal to one.

The vector of fragility model parameters is:

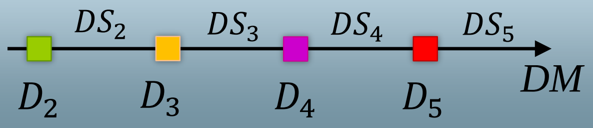
$\mathbf{\theta} = \{\alpha_0, \alpha_1\}$ for each level D_j

`theta_basic_modelk`
`k=1,2,3`

01 Tutorial

Hierarchical and Basic Fragility Modelling using MLE by MATLAB ToolBox – Parameter estimation

Building class 2 (Timber residential) of South Pacific 2009 Tsunami



Reese, S., Bradley, B. A., Bind, J., Smart, G., Power, W., and Sturman, J.: Empirical building fragilities from observed damage in the 2009 South Pacific tsunami, *Earth-Sci. Rev.*, 107(1-2), 156-173, 2011.

Hierarchical Fragility Modelling using Bayesian Inference

$$\pi_{ij} = P\left(D \geq D_{j+1} \middle| D \geq D_j, IM_i\right)$$

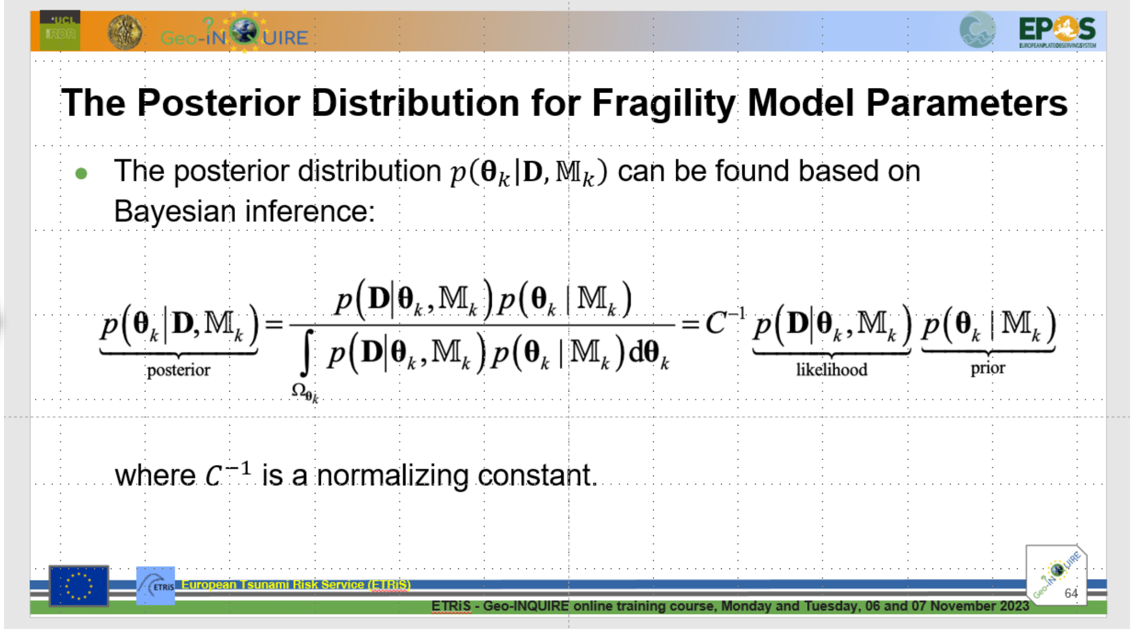
`posteriorFragilityFunction.m`

The adaptive MCMC procedure for drawing samples from the joint posterior $p(\boldsymbol{\theta}_k|\mathbf{D}, \mathbb{M}_k)$ of $\boldsymbol{\theta}_k$ given model $\mathbb{M}_k$ is carried out by considering 6 chains (simulation levels), and a maximum of 2000 samples per chain.

Samples $\{\boldsymbol{\theta}_{k,1}, \boldsymbol{\theta}_{k,2}, \cdots, \boldsymbol{\theta}_{k,N_d}\}$

$$\boldsymbol{\theta} = \left[\{\alpha_{0,j}, \alpha_{1,j}\}, j = 0{:}\,N_{DS} - 1\right]$$

`sample_theta_modelk`
`k=1,2,3`

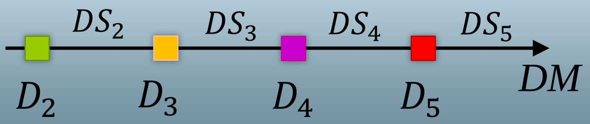


Prior: A multivariate normal distribution with zero correlation between the pairs of model parameters $\boldsymbol{\theta}_k$

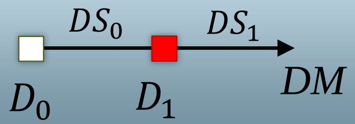
02 **Tutorial**

Hierarchical Fragility Modelling using Bayesian Inference by ComputeFrag - Parameter estimation

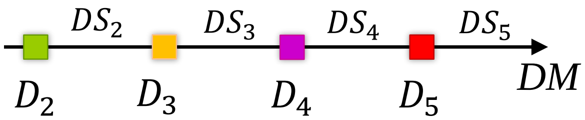
Building class 2 (Timber residential) of South Pacific 2009 Tsunami



Mixed Building type (Masonry and Wood) of Dichato, Maule, 2010 Chile Tsunami

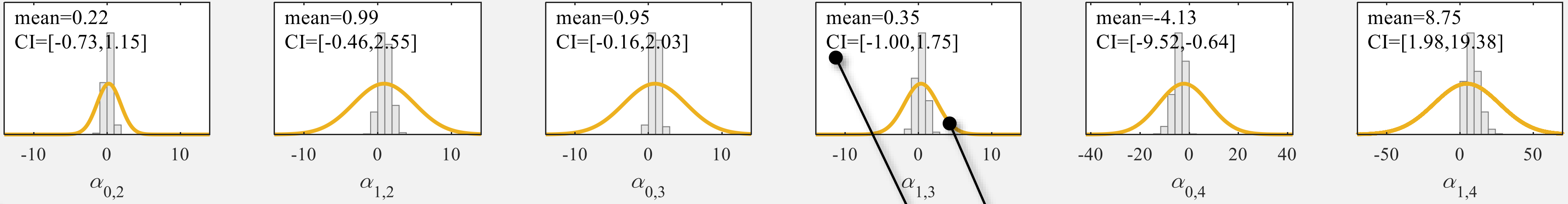


Mas, E., Koshimura, S., Suppasri, A., Matsuoka, M., Matsuyama, M., Yoshii, T., Jimenez, C., Yamazaki, F. and Imamura, F., 2012. Developing Tsunami fragility curves using remote sensing and survey data of the 2010 Chilean Tsunami in Dichato. *Natural Hazards and Earth System Sciences*, 12(8), pp.2689-2697.



Hierarchical Fragility Modelling using Bayesian Inference – Parameter estimation

sample_theta_model3



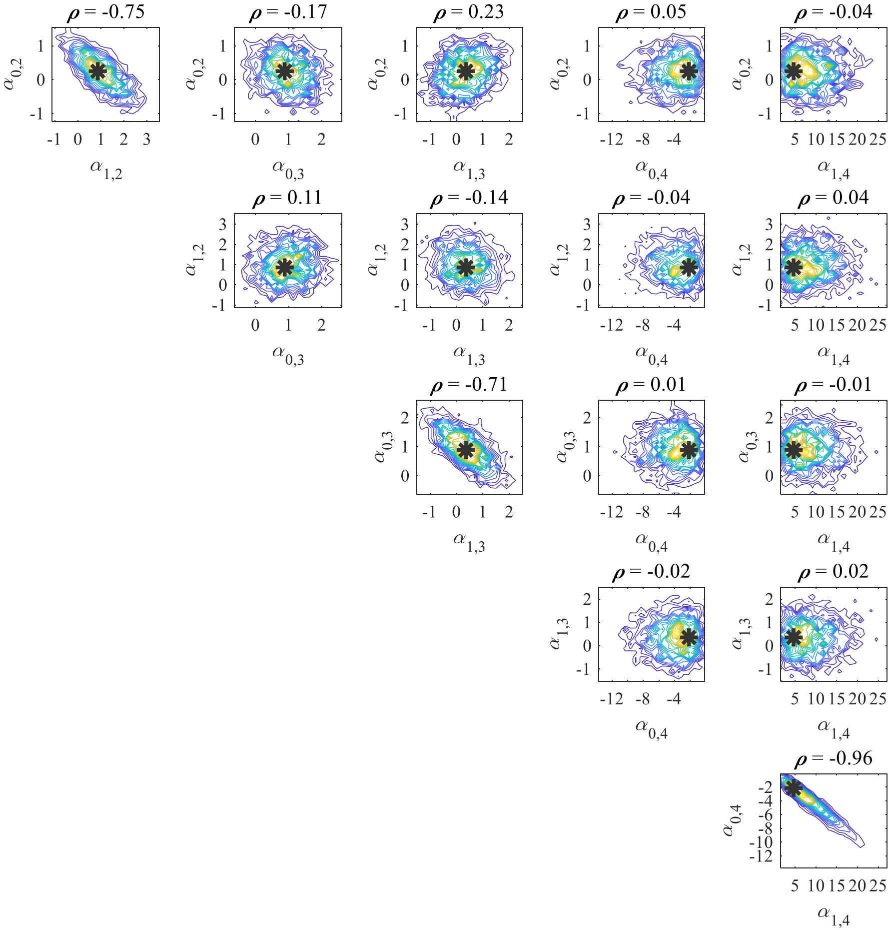
The marginal normal priors with large COV's

CI: 2% and 98% of the data based on the counted statistics

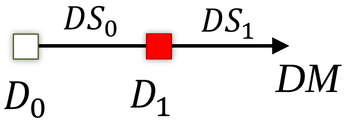
theta_prior_model3

0.2511
0.8625
0.8828
0.3553
-2.1409
4.6477

Hierarchical Fragility Modelling using MLE – Parameter estimation

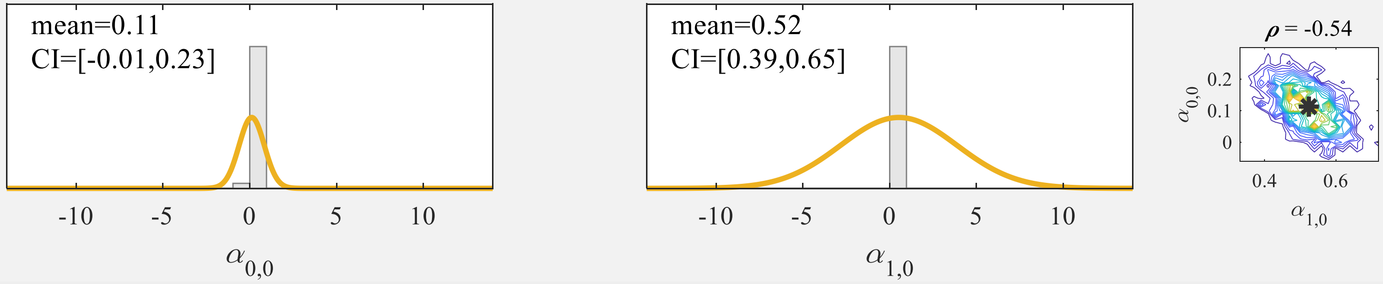


ρ = -0.75
ρ = -0.17
ρ = 0.23
ρ = 0.05
ρ = -0.04
ρ = 0.11
ρ = -0.14
ρ = -0.04
ρ = 0.04
ρ = -0.71
ρ = 0.01
ρ = -0.01
ρ = -0.02
ρ = 0.02
ρ = -0.96



Hierarchical Fragility Modelling using Bayesian Inference – Parameter estimation

sample_theta_model2



theta_prior_model2

0.1136
0.5244

Hierarchical Fragility Modelling using MLE – Parameter estimation

Hierarchical Fragility Modelling using MLE of fragility model parameters

$$\boldsymbol{\theta} = \left[\{\alpha_{0,j}, \alpha_{1,j}\}, j = 0{:}N_{DS} - 1\right]$$

`calculate_fragility.m`

Nat. Hazards Earth Syst. Sci., 23, 909–931, 2023
https://doi.org/10.5194/nhess-23-909-2023

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Empirical tsunami fragility modelling for hierarchical damage levels

Fatemeh Jalayer[1,2], Hossein Ebrahimian[2], Konstantinos Trevlopoulos[2], and Brendon Bradley[3]

[1]Institute for Risk and Disaster Reduction, University College London, Gower Street, London WC1E 6BT, UK
[2]Department of Structures for Engineering and Architecture, University of Naples Federico II, Naples 80125, Italy
[3]Department of Civil and Natural Resources Engineering, University of Canterbury, Private Bag 4800, Christchurch 8140, New Zealand

Fragility Curve based on the vector of *IM* defined

`FA_modelk, k=1,2,3`

Fragility Assessment using Generalized Regression

- The probabilities of being in different damage states can be calculated in a recursive way:

$$P(DS_j|IM_i) = \begin{cases} (1-\pi_{ij})\cdot\left[1-\sum_{k=0}^{j-1} P(DS_k|IM_i)\right] & \text{for } j \geq 1 \\ 1-\pi_{i0} \triangleq P(D<D_1|IM_i) & \text{for } j=0 \end{cases}$$

$$P(DS_{N_{DS}}|IM_i) = P(D \geq D_{N_{DS}}|IM_i) = 1-\sum_{j=0}^{N_{DS}-1} P(DS_j|IM_i)$$

$$P(D \geq D_j|IM_i) = P(DS_j|IM_i) + P(D \geq D_{j+1}|IM_i) \quad \text{for } 0 \leq j < N_{DS}$$

Basic Fragility Modelling using MLE of the fragility model parameters
$\boldsymbol{\theta} = \{\alpha_0, \alpha_1\}$ for each level D_j

`calculate_fragility_basic.m`

The fragility $\pi_{ij} = P(D \geq D_j | IM_i)$ is obtained by using a generalized linear regression model according with "logit", "probit" or "cloglog" link function fitted to the damage data ($\mathbb{M}_k$ where $k = 1{:}3$)

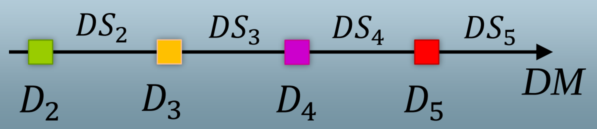
Fragility Curve based on the vector of *IM* defined

`FA_basic_modelk`
`k=1,2,3`

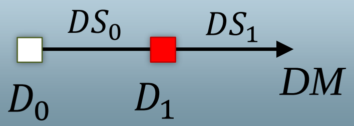
03 Tutorial

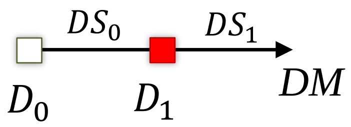
Hierarchical and Basic Fragility Assessment using MLE of fragility model parameters

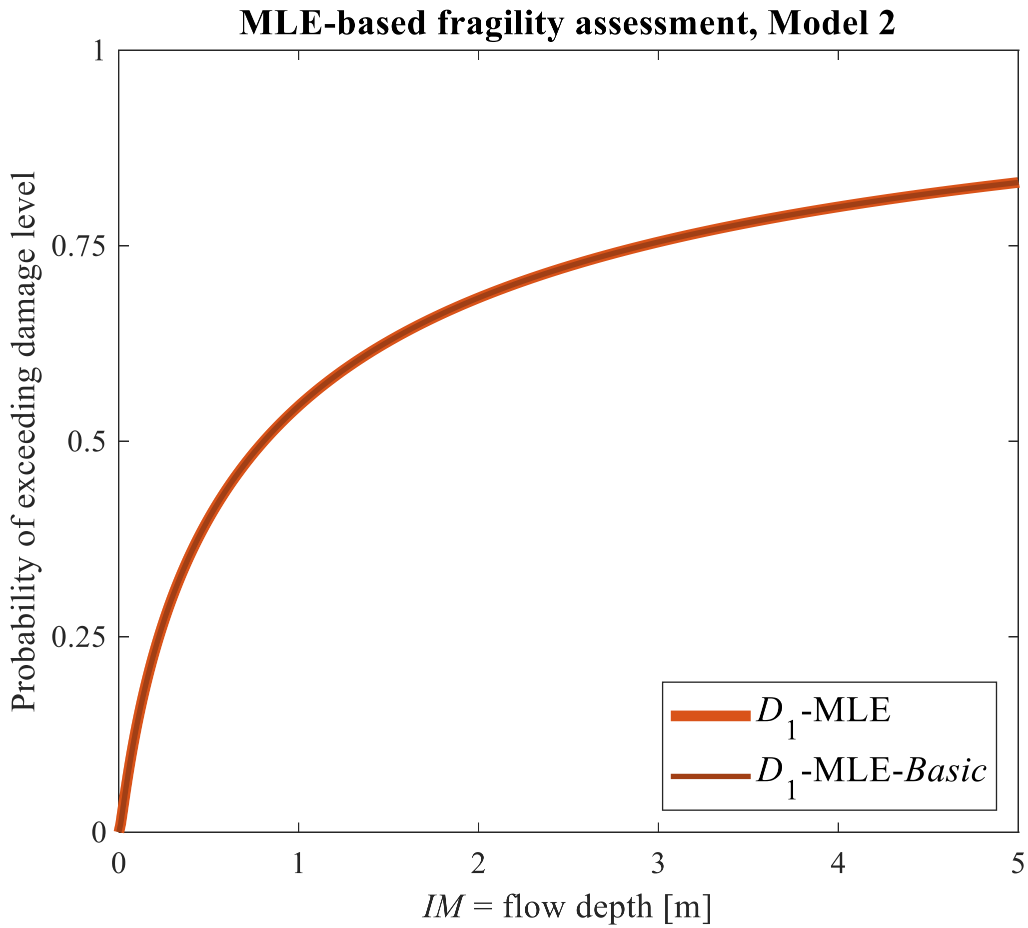
Building class 2 (Timber residential) of South Pacific 2009 Tsunami

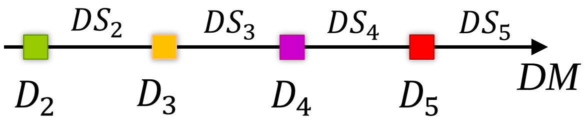


Mixed Building type (Masonry and Wood) of Dichato, Maule, 2010 Chile Tsunami

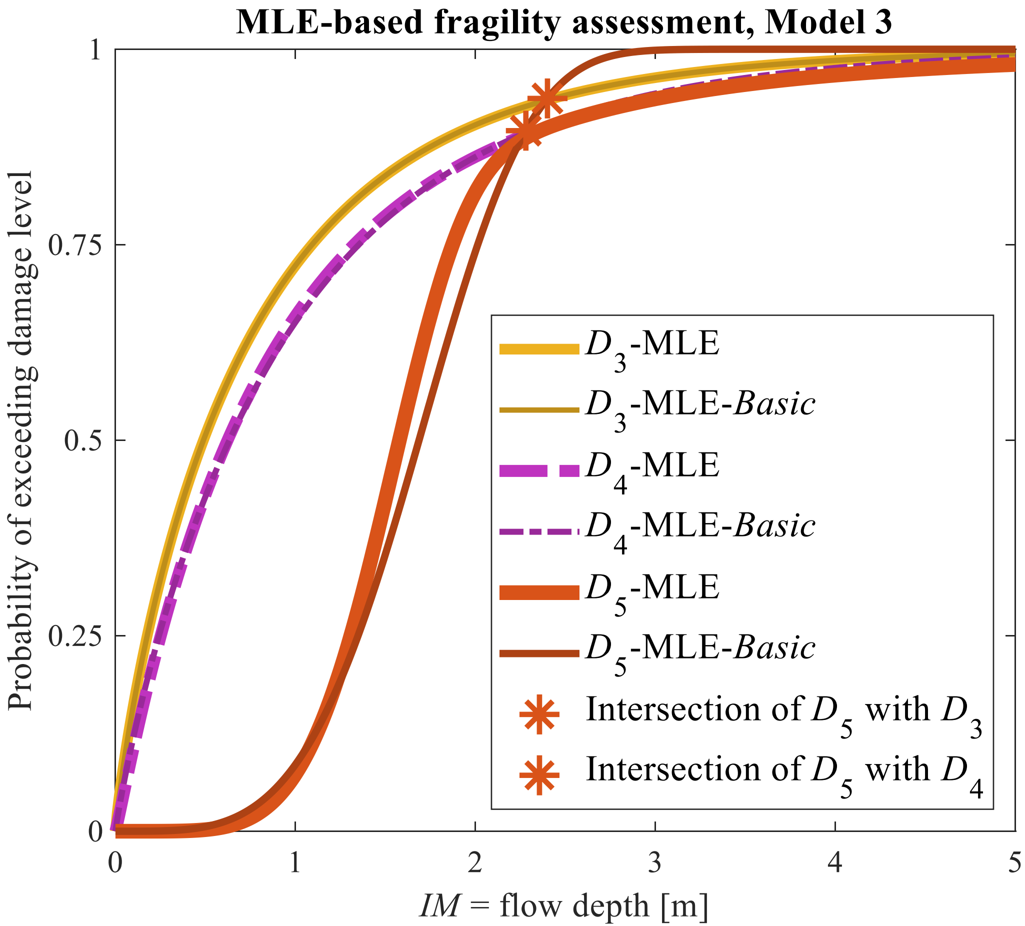



DS_0
DS_1
D_0
D_1
DM


MLE-based fragility assessment, Model 2
Probability of exceeding damage level
1
0.75
0.5
0.25
0
0
1
2
3
4
5
IM = flow depth [m]
D_1-MLE
D_1-MLE-*Basic*



DS_2
DS_3
DS_4
DS_5
D_2
D_3
D_4
D_5
DM



MLE-based fragility assessment, Model 3
Probability of exceeding damage level
0
0.25
0.5
0.75
1
0
1
2
3
4
5
IM = flow depth [m]
D_3-MLE
D_3-MLE-*Basic*
D_4-MLE
D_4-MLE-*Basic*
D_5-MLE
D_5-MLE-*Basic*
Intersection of D_5 with D_3
Intersection of D_5 with D_4

RobustFragility_DS.m

Hierarchical Fragility Modelling using Bayesian Inference for the fragility model parameters

$$\{\boldsymbol{\theta}_{k,1},\boldsymbol{\theta}_{k,2},\cdots,\boldsymbol{\theta}_{k,N_d}\}$$

$$\boldsymbol{\theta}=[\{\alpha_{0,j},\alpha_{1,j}\},j=0:N_{DS}-1]$$

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Fragility Curve and its confidence interval based on the vector of *IM*

RF_modelk, k=1,2,3

Robust Fragility Assessment

- Robust Fragility (RF) is defined as the expected value for a prescribed fragility model considering the joint probability distribution for the fragility model parameters $\boldsymbol{\theta}_k$. The RF herein can be expressed as:

$$P(D\ge D_j|IM,\mathbf{D},\mathbb{M}_k)=\int_{\Omega_{\boldsymbol{\theta}_k}} P(D\ge D_j|IM,\boldsymbol{\theta}_k)\,p(\boldsymbol{\theta}_k|\mathbf{D},\mathbb{M}_k)\,d\boldsymbol{\theta}_k=\mathbb{E}_{\boldsymbol{\theta}_k|\mathbf{D},\mathbb{M}_k}\left[P(D\ge D_j|IM,\boldsymbol{\theta}_k)\right]$$

$$\sigma^2_{\boldsymbol{\theta}_k|\mathbf{D},\mathbb{M}_k}\left[P(D\ge D_j|IM,\boldsymbol{\theta}_k)\right]=\underbrace{\mathbb{E}_{\boldsymbol{\theta}_k|\mathbf{D},\mathbb{M}_k}\left[P(D\ge D_j|IM,\boldsymbol{\theta}_k)^2\right]}_{\cong\frac{1}{N_d}\sum_{i=1}^{N_d}P(D\ge D_j|IM,\boldsymbol{\theta}_{k,i})^2}-\underbrace{\left(\mathbb{E}_{\boldsymbol{\theta}_k|\mathbf{D},\mathbb{M}_k}\left[P(D\ge D_j|IM,\boldsymbol{\theta}_k)\right]\right)^2}_{=P(D\ge D_j|IM,\mathbf{D},\mathbb{M}_k)^2}\quad\text{(Eq.16)}$$

RobustFragility_DS.m

Hierarchical Fragility Modelling using Bayesian Inference for the fragility model parameters

$$\{\boldsymbol{\theta}_{k,1}, \boldsymbol{\theta}_{k,2}, \cdots, \boldsymbol{\theta}_{k,N_d}\}$$

$$\boldsymbol{\theta} = [\{\alpha_{0,j}, \alpha_{1,j}\}, j = 0 : N_{DS} - 1]$$

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[3]Department of Civil and Natural Resources Engineering, University of Canterbury, Private Bag 4800, Christchurch 8140, New Zealand

Fragility Curve and its confidence interval based on the vector of *IM*

RF_modelk, k=1,2,3

Geo-INQUIRE

Using Monte Carlo Simulation for Fragility Assessment

- The RF integral can be solved numerically by employing Monte Carlo simulation with N_d generated samples from the vector $\boldsymbol{\theta}_k$ as follows:

$$P\left(D \geq D_j \middle| IM, \mathbf{D}, \mathbb{M}_k\right) \cong \frac{1}{N_d} \sum_{l=1}^{N_d} P\left(D \geq D_j \middle| IM, \boldsymbol{\theta}_{k,l}\right)$$

European Tsunami Risk Service (ETRiS)
ETRiS - Geo-INQUIRE online training course, Monday and Tuesday, 06 and 07 November 2023
68

RobustFragility_DS.m

Hierarchical Fragility Modelling using Bayesian Inference for the fragility model parameters

$$\{\boldsymbol{\theta}_{k,1}, \boldsymbol{\theta}_{k,2}, \cdots, \boldsymbol{\theta}_{k,N_d}\}$$

$$\boldsymbol{\theta} = [\{\alpha_{0,j}, \alpha_{1,j}\}, j = 0{:}\, N_{DS} - 1]$$

Nat. Hazards Earth Syst. Sci., 23, 909–931, 2023
https://doi.org/10.5194/nhess-23-909-2023

Natural Hazards and Earth System Sciences
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Empirical tsunami fragility modelling for hierarchical damage levels

Fatemeh Jalayer[1,2], Hossein Ebrahimian[2], Konstantinos Trevlopoulos[2], and Brendon Bradley[3]
[1]Institute for Risk and Disaster Reduction, University College London, Gower Street, London WC1E 6BT, UK
[2]Department of Structures for Engineering and Architecture, University of Naples Federico II, Naples 80125, Italy
[3]Department of Civil and Natural Resources Engineering, University of Canterbury, Private Bag 4800, Christchurch 8140, New Zealand

Fragility Curve and its confidence interval based on the vector of *IM*

RF_modelk, k=1,2,3

Using Monte Carlo Simulation for Fragility Assessment

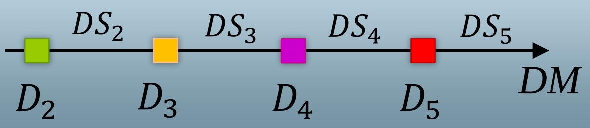
- The integral equation for standard deviation of the fragility can be solved numerically by employing Monte Carlo simulation with N_d generated samples from the vector $\boldsymbol{\theta}_k$ as follows:

$$\sigma^2_{\boldsymbol{\theta}_k|\mathbf{D},\mathbb{M}_k}\left[P\left(D \geq D_j \middle| IM, \boldsymbol{\theta}_k\right)\right] \cong \frac{1}{N_d}\sum_{i=1}^{N_d} P\left(D \geq D_j \middle| IM, \boldsymbol{\theta}_{k,l}\right)^2 - P\left(D \geq D_j \middle| IM, \mathbf{D}, \mathbb{M}_k\right)^2$$

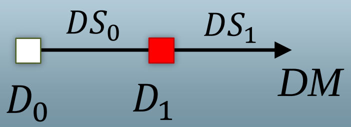
04 Tutorial

Hierarchical Fragility Assessment using Bayesian Inference

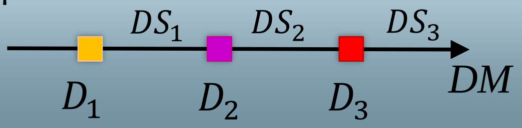
Building class 2 (Timber residential) of South Pacific 2009 Tsunami



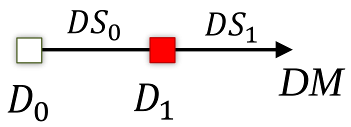
Mixed Building type (Masonry and Wood) of Dichato, Maule, 2010 Chile Tsunami



Building class 3 (Non engineered light timber) of Sulawesi-Palu 2018 Tsunami



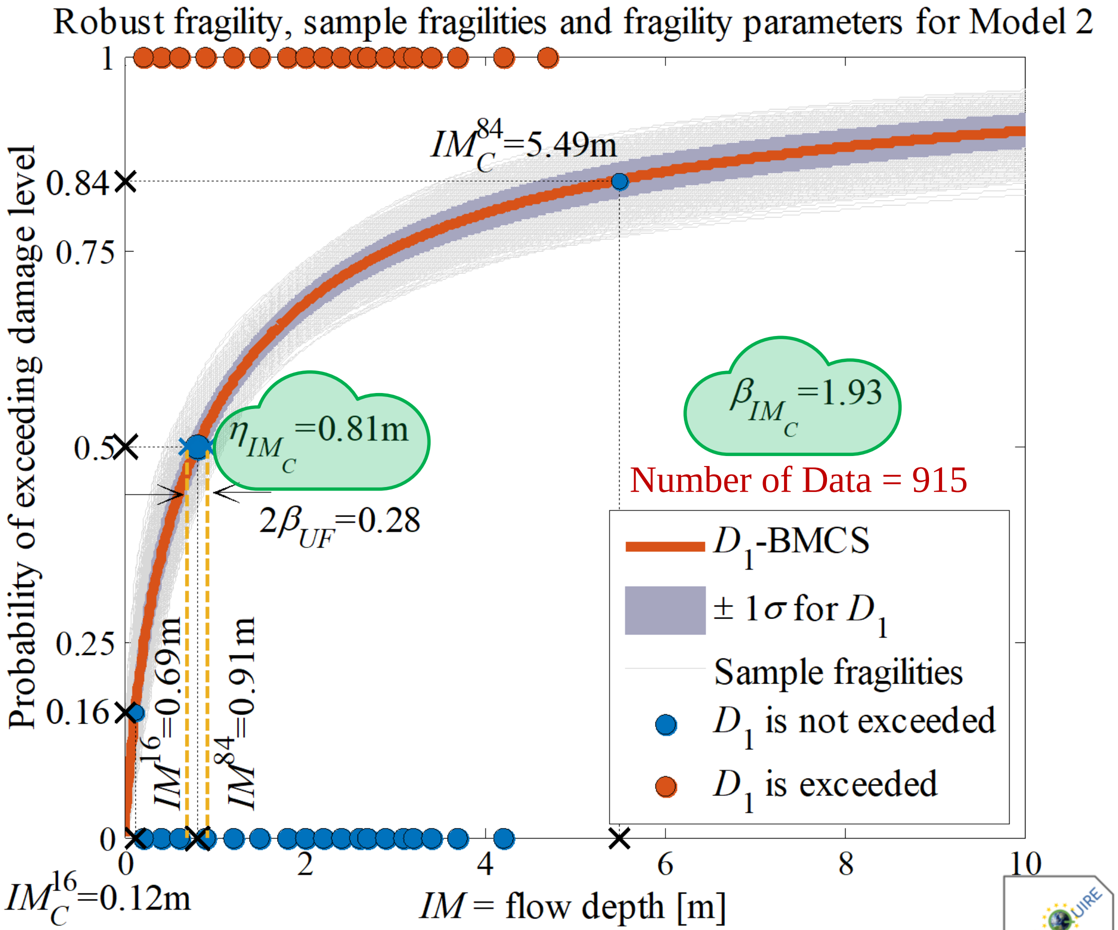
Paulik, R., Gusman, A., Williams, J. H. et al. (2019). Tsunami hazard and built environment damage observations from Palu city after the September 28 2018 Sulawesi earthquake and tsunami. Pure Appl. Geophys. 176, 3305-3321.

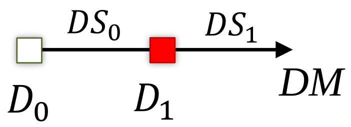


The median intensity, η_{IM_C}, for a given damage level, is calculated as the *IM* corresponding to 50% probability on the fragility curve.

The logarithmic standard deviation (dispersion) of the equivalent lognormal fragility curve at the onset of damage threshold, β_{IM_C}, is estimated as half of the logarithmic distance between the IMs corresponding to the probabilities of 16% (IM_C^{16}) and the 84% (IM_C^{84}) on the fragility curve; thus, the dispersion can be estimated as

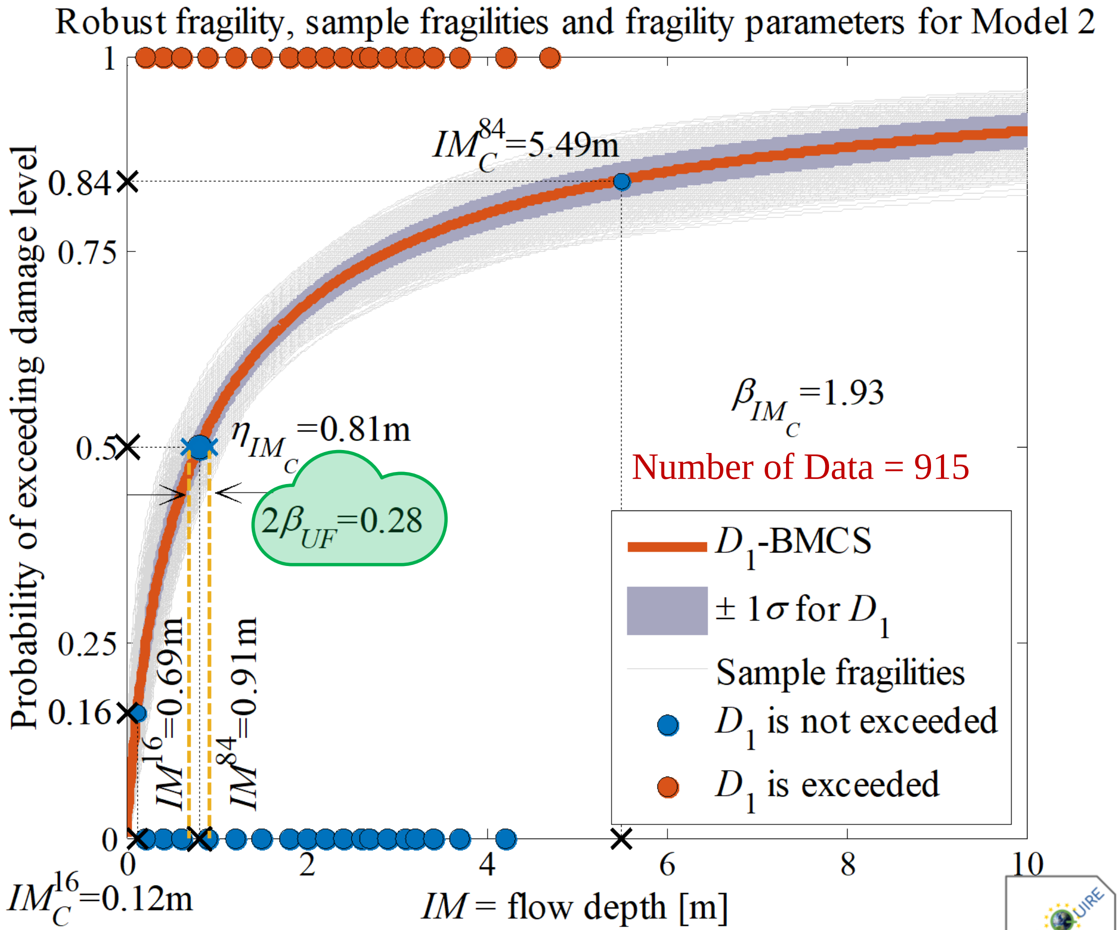
$\beta_{IM_C} = 0.50 \times \ln\left(IM_C^{84}/IM_C^{16}\right)$.

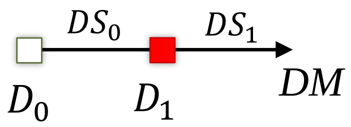


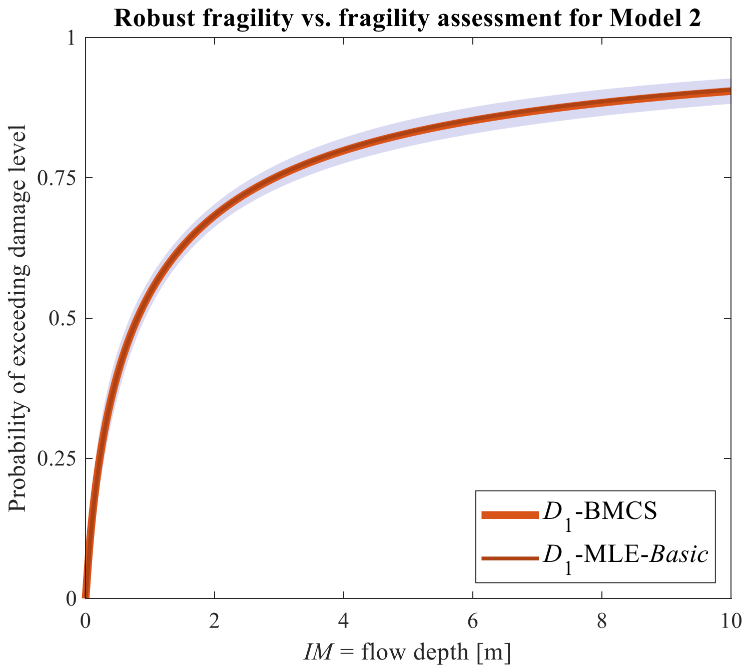


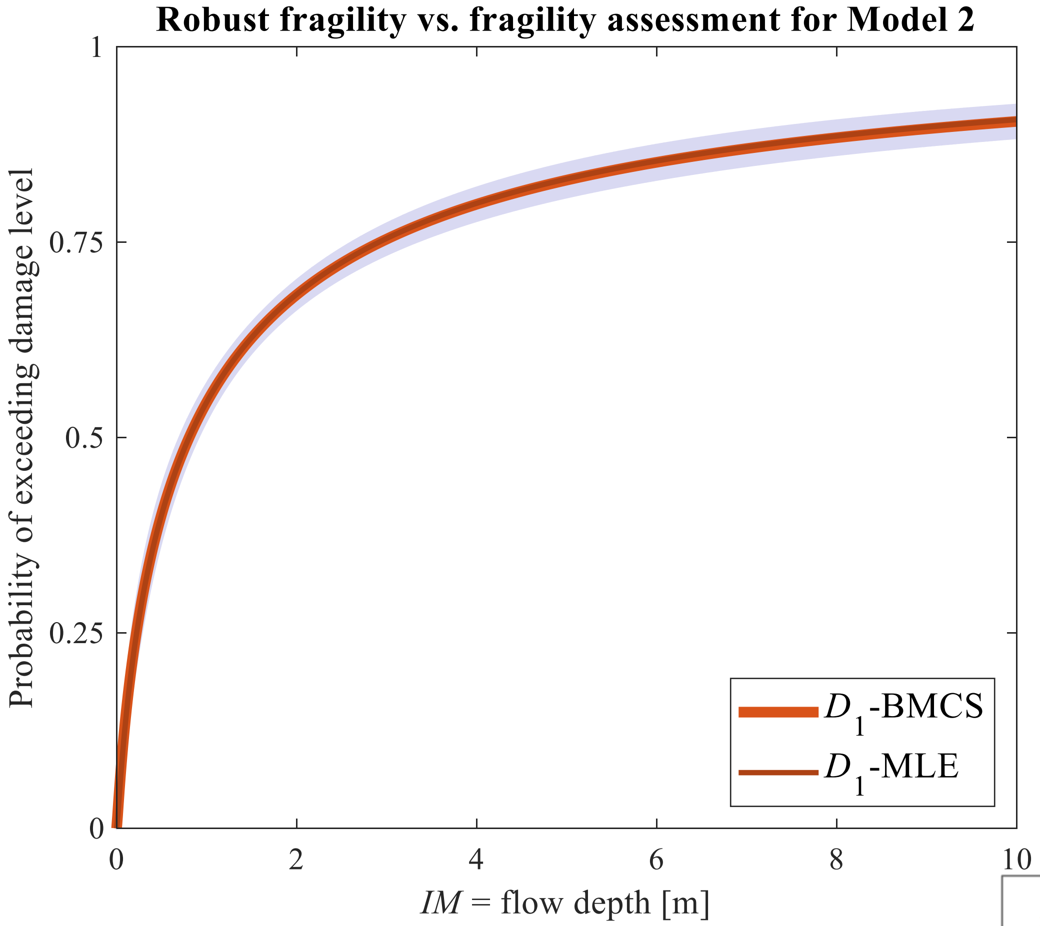
The overall effect of epistemic uncertainties (due to the uncertainty in the fragility model parameters and reflecting the effect of limited sample size) on the median of the empirical fragility curve is considered through (logarithmic) intensity-based standard deviation denoted as β_{UF}. It can be estimated as half of the (natural) logarithmic distance (along the *IM* axis) between the median intensities (i.e., 50% probability) of the RF's derived with 16% (denoted as IM^{84}) and 84% (IM^{16}) confidence levels, respectively:

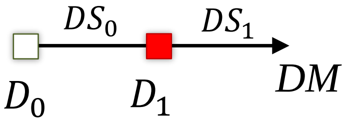
$\beta_{UF} = 0.50 \times \ln(IM^{84}/IM^{16})$.




DS_0
DS_1
D_0
D_1
DM

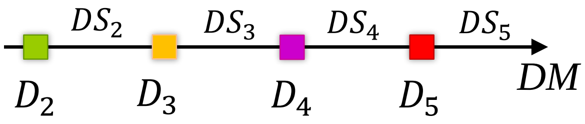

Robust fragility vs. fragility assessment for Model 2
Probability of exceeding damage level
0
0.25
0.5
0.75
1
0
2
4
6
8
10
IM = flow depth [m]
D_1-BMCS
D_1-MLE-*Basic*


Robust fragility vs. fragility assessment for Model 2
Probability of exceeding damage level
0
0.25
0.5
0.75
1
0
2
4
6
8
10
IM = flow depth [m]
D_1-BMCS
D_1-MLE



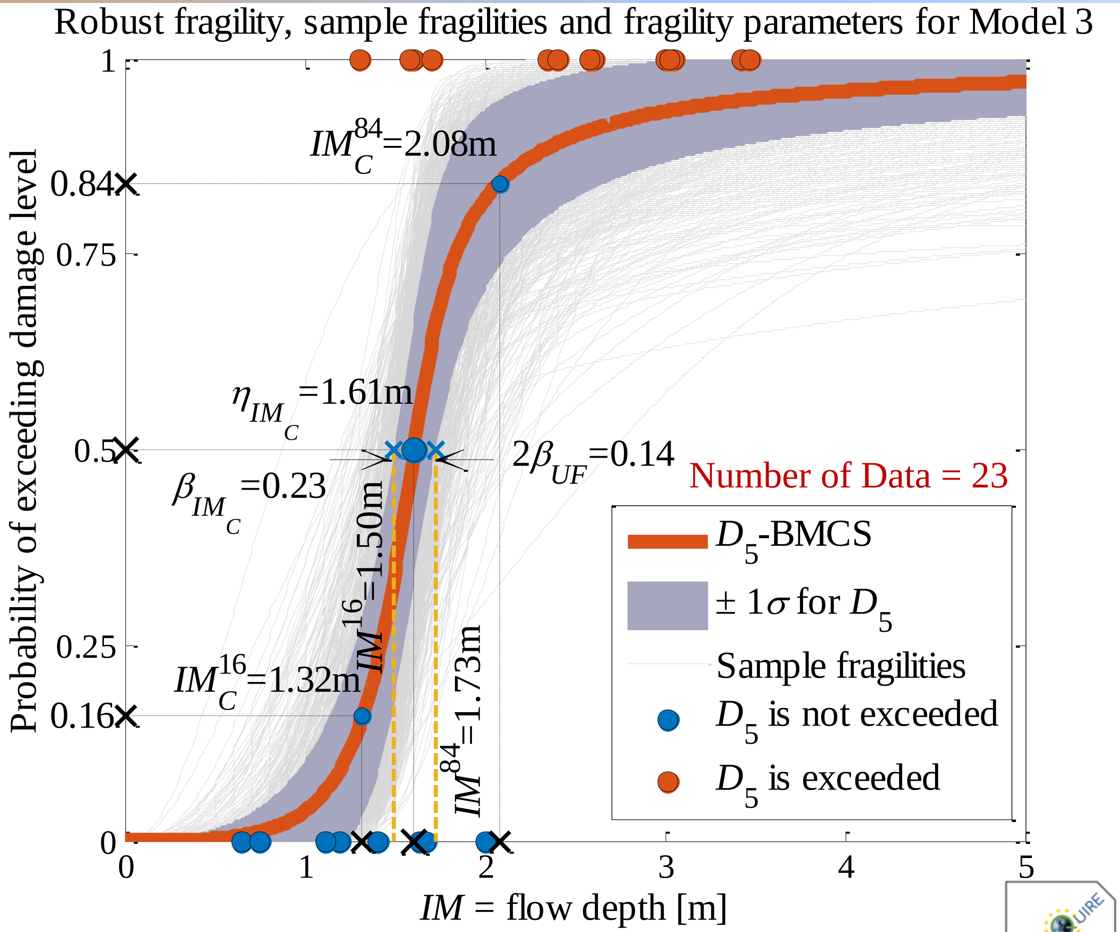
Jupyter Notebooks for Fragility Visualisation

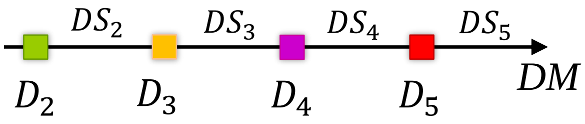
https://github.com/eurotsunamirisk/VisualizeFragility

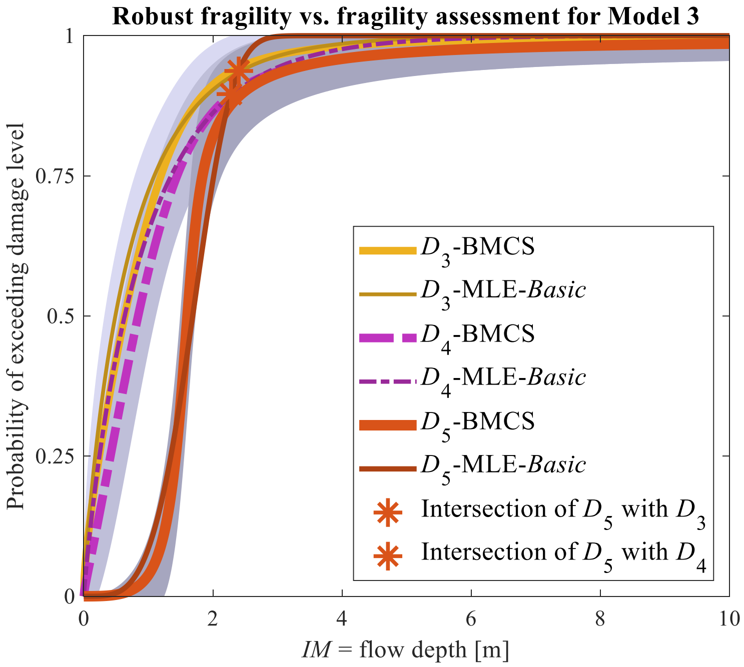


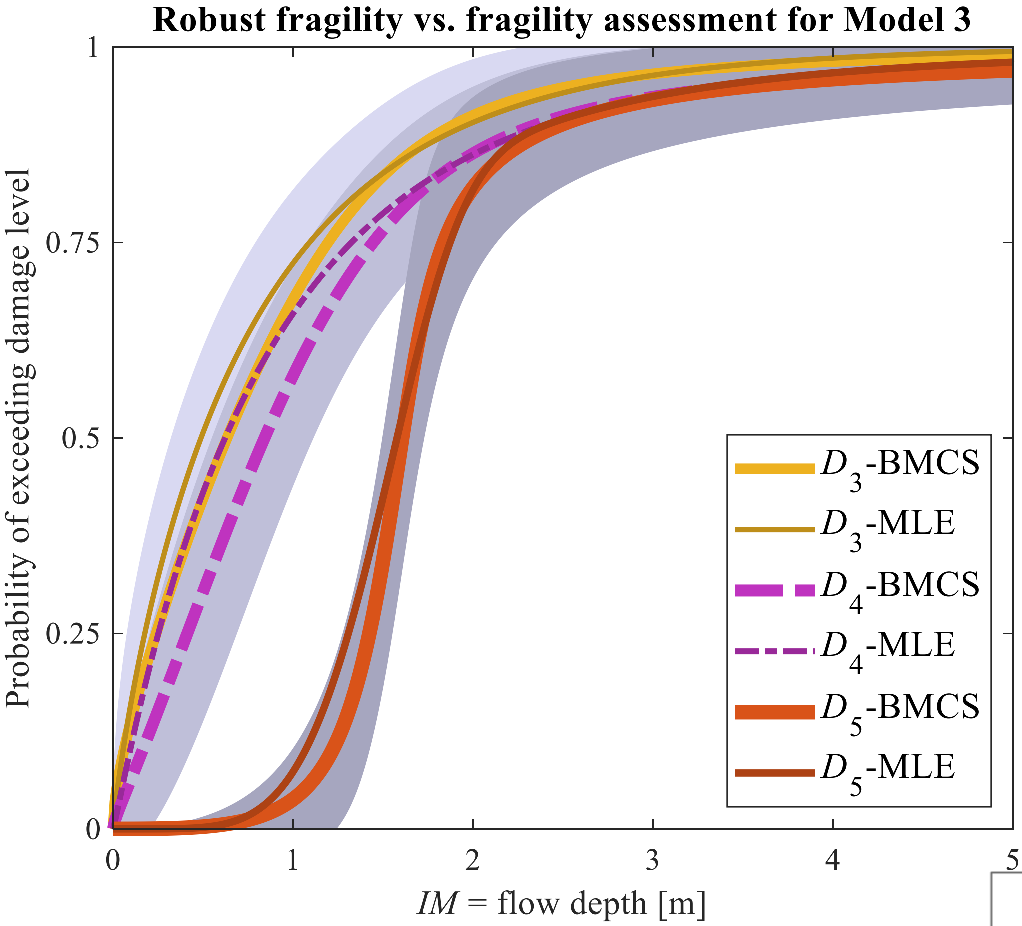
The Issue of rejected samples

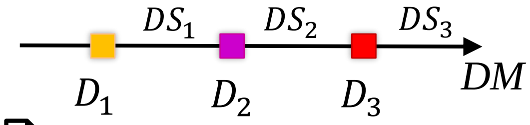
Some samples $\boldsymbol{\theta}_{k,l}$ may lead to fragility curves with unrealistic configurations as follows: (1) having negative slope as the *IM* increases; (2) Having high exceedance probability at very low *IM* values. To this end, those samples should be rejected. This case is more often when limited number of observed damages exists for a specific class of building.




DS_2
DS_3
DS_4
DS_5
D_2
D_3
D_4
D_5
DM


Robust fragility vs. fragility assessment for Model 3
Probability of exceeding damage level
IM = flow depth [m]
D_3-BMCS
D_3-MLE-*Basic*
D_4-BMCS
D_4-MLE-*Basic*
D_5-BMCS
D_5-MLE-*Basic*
Intersection of D_5 with D_3
Intersection of D_5 with D_4

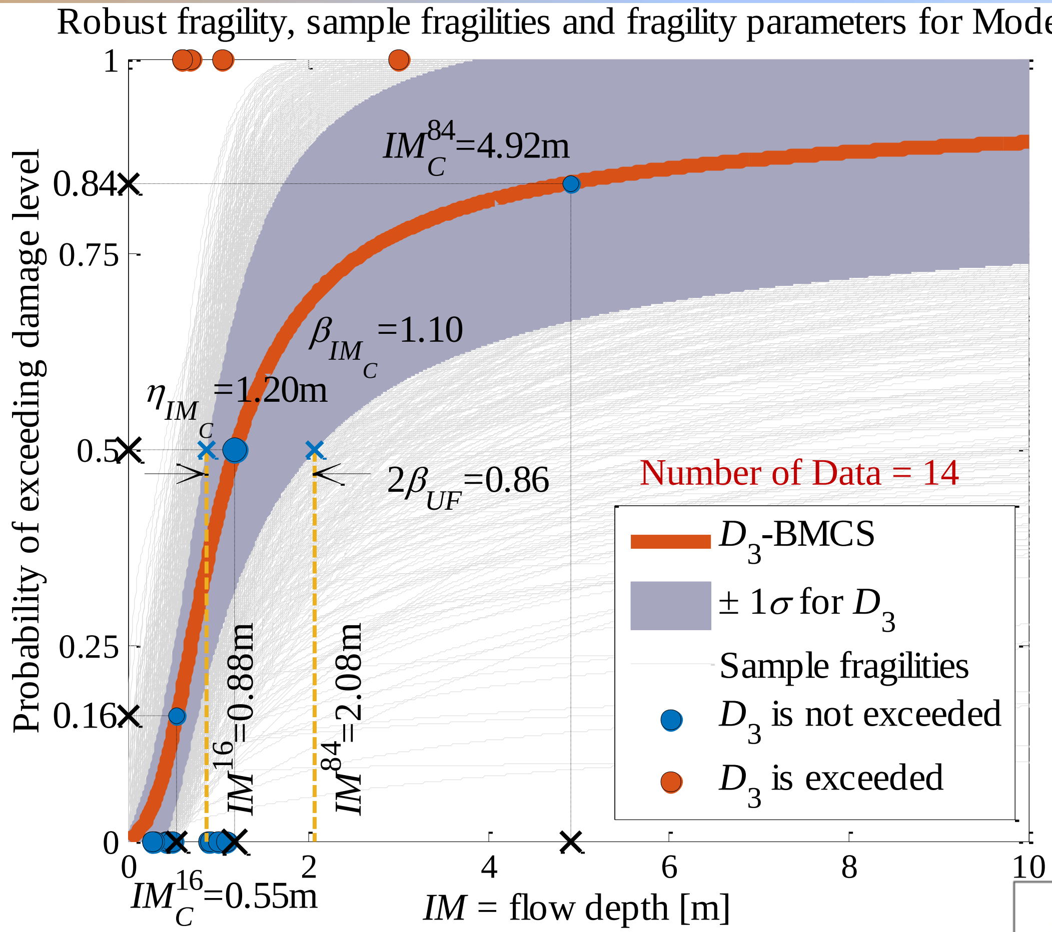

Robust fragility vs. fragility assessment for Model 3
Probability of exceeding damage level
IM = flow depth [m]
D_3-BMCS
D_3-MLE
D_4-BMCS
D_4-MLE
D_5-BMCS
D_5-MLE

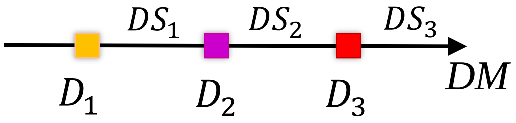


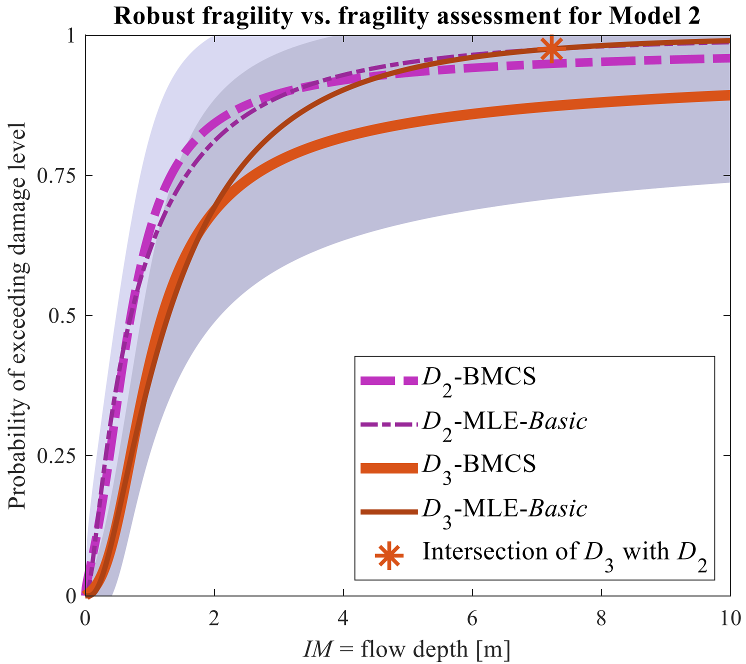
RF_model2

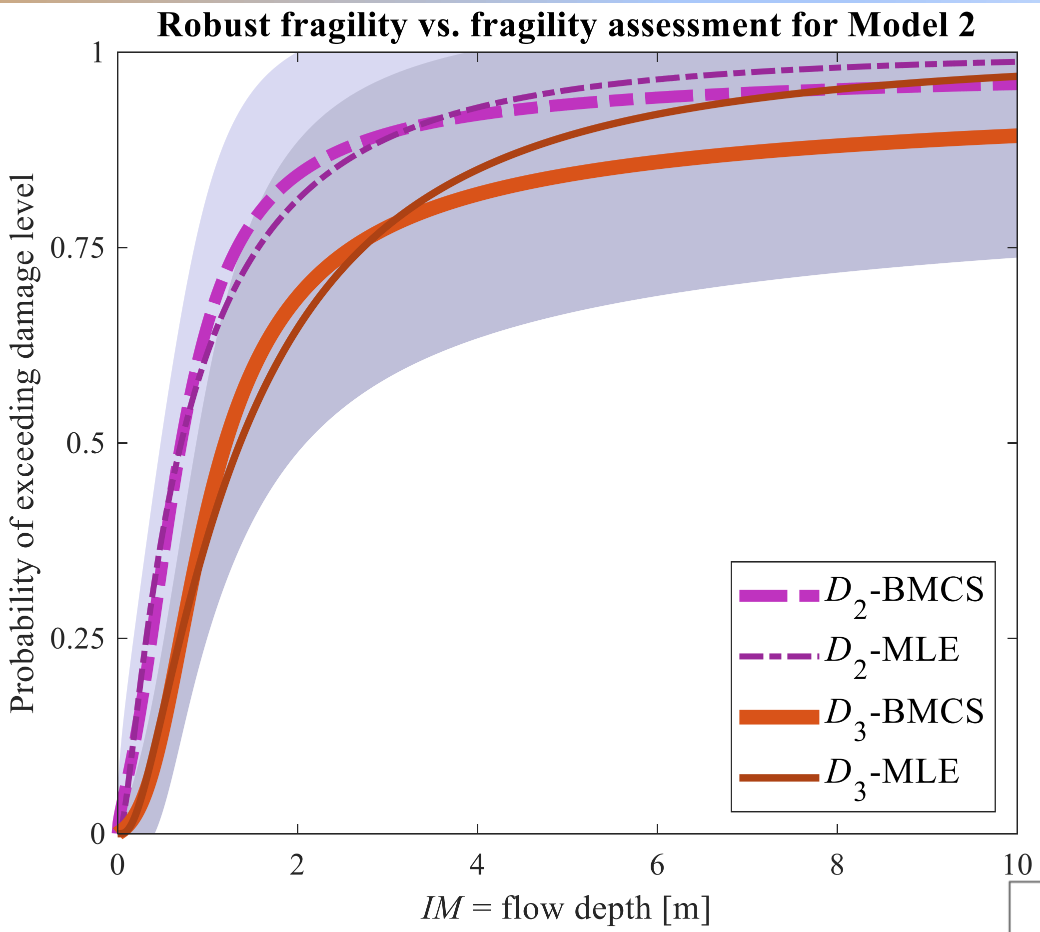
1x1 struct with 18 fields

Field	Value
fragility	1001x2 double
sfragility	1001x2 double
sample_fragility	1x1241 cell
sample_fragility_DSj	1x2 cell
rejected_samples	1x496 double
sample_theta	4x1241 double
RFp	1001x2 double
RFm	1001x2 double
RF84	1001x2 double
RF16	1001x2 double
etaIMc	[0.7062,1.1982]
IMc16	[0.2546,0.5475]
IMc84	[1.9800,4.9190]
etaIMc_RF84	[0.4847,0.8809]
etaIMc_RF16	[1.0361,2.0832]
betaIMc	[1.0256,1.0977]
betaUF	[0.3798,0.4304]
sample_PDS_IM	1x1241 cell




DS_1
DS_2
DS_3
D_1
D_2
D_3
DM


Robust fragility vs. fragility assessment for Model 2
Probability of exceeding damage level
1
0.75
0.5
0.25
0
0
2
4
6
8
10
IM = flow depth [m]
D_2-BMCS
D_2-MLE-*Basic*
D_3-BMCS
D_3-MLE-*Basic*
Intersection of D_3 with D_2


Robust fragility vs. fragility assessment for Model 2
Probability of exceeding damage level
1
0.75
0.5
0.25
0
0
2
4
6
8
10
IM = flow depth [m]
D_2-BMCS
D_2-MLE
D_3-BMCS
D_3-MLE

Bayesian model class selection (BMCS) for identifying the best link model to use in the generalized linear regression scheme

`calculate_logE.m`

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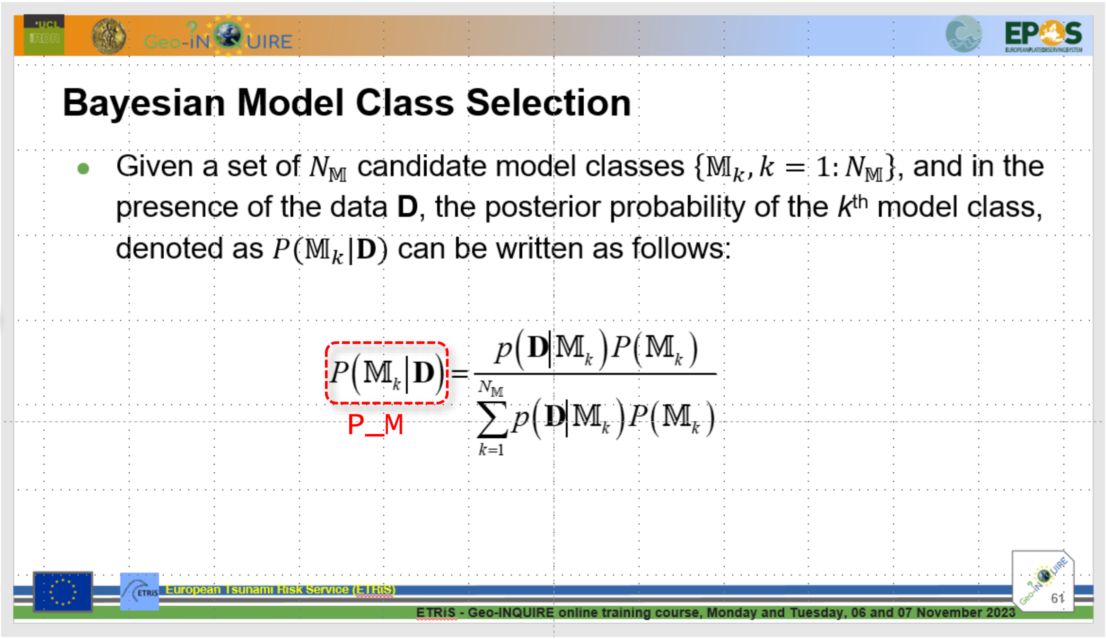
[2]Department of Structures for Engineering and Architecture, University of Naples Federico II, Naples 80125, Italy
[3]Department of Civil and Natural Resources Engineering, University of Canterbury, Private Bag 4800, Christchurch 8140, New Zealand

Posterior probability of each model class, $P(\mathbb{M}_k|\mathbf{D})$, the log evidence, $\ln[p(\mathbf{D}|\mathbb{M}_k)]$, and the two terms

`P_M`, `log_evidence`, `meanlogLikelihood`, `meanlogratioP`



Fragility Model Class Selection using Bayesian Inference

Bayesian model class selection (BMCS) for identifying the best link model to use in the generalized linear regression scheme

`calculate_logE.m`

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Posterior probability of each model class, $P(\mathbb{M}_k|\mathbf{D})$, the log evidence, $\ln[p(\mathbf{D}|\mathbb{M}_k)]$, and the two terms

`P_M`, `log_evidence`, `meanlogLikelihood`, `meanlogratioP`

The (log) evidence

- that logarithm of the evidence (called *log-evidence*) $\ln[p(\mathbf{D}|\mathbb{M}_k)]$ can be written as:

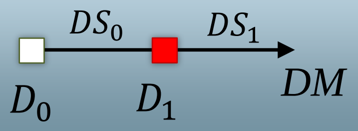
$$\underbrace{\boxed{\ln\left[p(\mathbf{D}|\mathbb{M}_k)\right]}}_{\texttt{log\_evidence}} = \underbrace{\int_{\Omega_{\theta_k}} \ln\left[p(\mathbf{D}|\boldsymbol{\theta}_k,\mathbb{M}_k)\right] p(\boldsymbol{\theta}_k|\mathbf{D},\mathbb{M}_k)\,d\boldsymbol{\theta}_k}_{\boxed{Term\ 1}\ \texttt{meanlogLikelihood}} - \underbrace{\int_{\Omega_{\theta_k}} \ln\left[\frac{p(\boldsymbol{\theta}_k|\mathbf{D},\mathbb{M}_k)}{p(\boldsymbol{\theta}_k|\mathbb{M}_k)}\right] p(\boldsymbol{\theta}_k|\mathbf{D},\mathbb{M}_k)\,d\boldsymbol{\theta}_k}_{\boxed{Term\ 2}\ \texttt{meanlogratioP}}$$

- "Term 1" denotes the posterior mean of the log-likelihood, which is a measure of the average data fit to model $\mathbb{M}_k$. "Term 2" is the relative entropy between the prior $p(\boldsymbol{\theta}_k|\mathbb{M}_k)$ and the posterior $p(\boldsymbol{\theta}_k|\mathbf{D},\mathbb{M}_k)$ of $\boldsymbol{\theta}_k$ given model $\mathbb{M}_k$, which is a measure of the distance between the two PDFs.

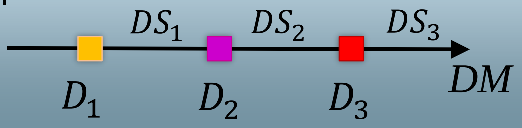
05 **Tutorial**

Bayesian model class selection (BMCS)

Mixed Building type (Masonry and Wood) of Dichato, Maule, 2010 Chile Tsunami



Building class 3 (Non engineered light timber) of Sulawesi-Palu 2018 Tsunami



```
------ The posterior probability of Model Class 1 is 0.323
------ The posterior probability of Model Class 2 is 0.342
------ The posterior probability of Model Class 3 is 0.335
```

Model Class	Term 1: Average Data Fit	Term 2: Information Gain	Log-Evidence	Posterior Probability of the model
$\mathbb{M}_1$	-545.208	5.4913	-550.700	0.3228
$\mathbb{M}_2$	-545.085	5.5575	-550.642	0.3419
$\mathbb{M}_3$	-544.257	6.405	-550.662	0.3353

```
------ The posterior probability of Model Class 1 is 0.210
------ The posterior probability of Model Class 2 is 0.570
------ The posterior probability of Model Class 3 is 0.220
```

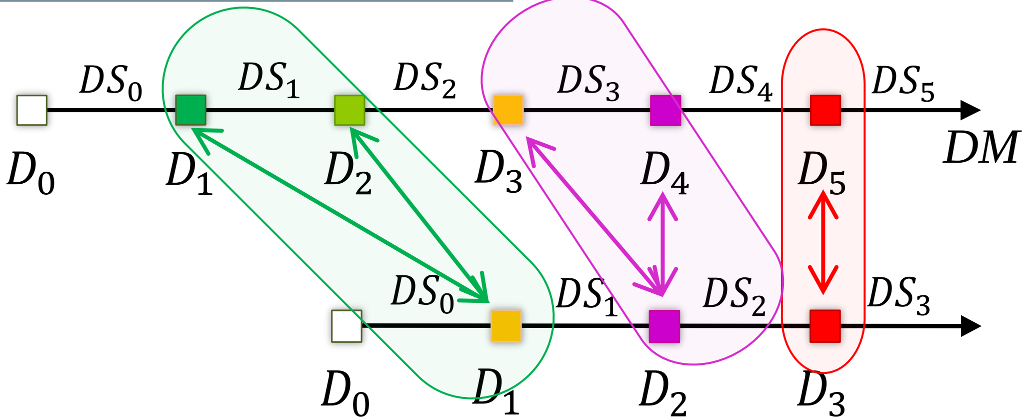
Model Class	Term 1: Average Data Fit	Term 2: Information Gain	Log-Evidence	Posterior Probability of the model
$\mathbb{M}_1$	-15.8034	4.2741	-20.0775	0.2101
$\mathbb{M}_2$	-15.1575	3.9226	-19.0802	0.5697
$\mathbb{M}_3$	-14.6294	5.4015	-20.0309	0.2202

Comparing fragility curves for similar classes of buildings

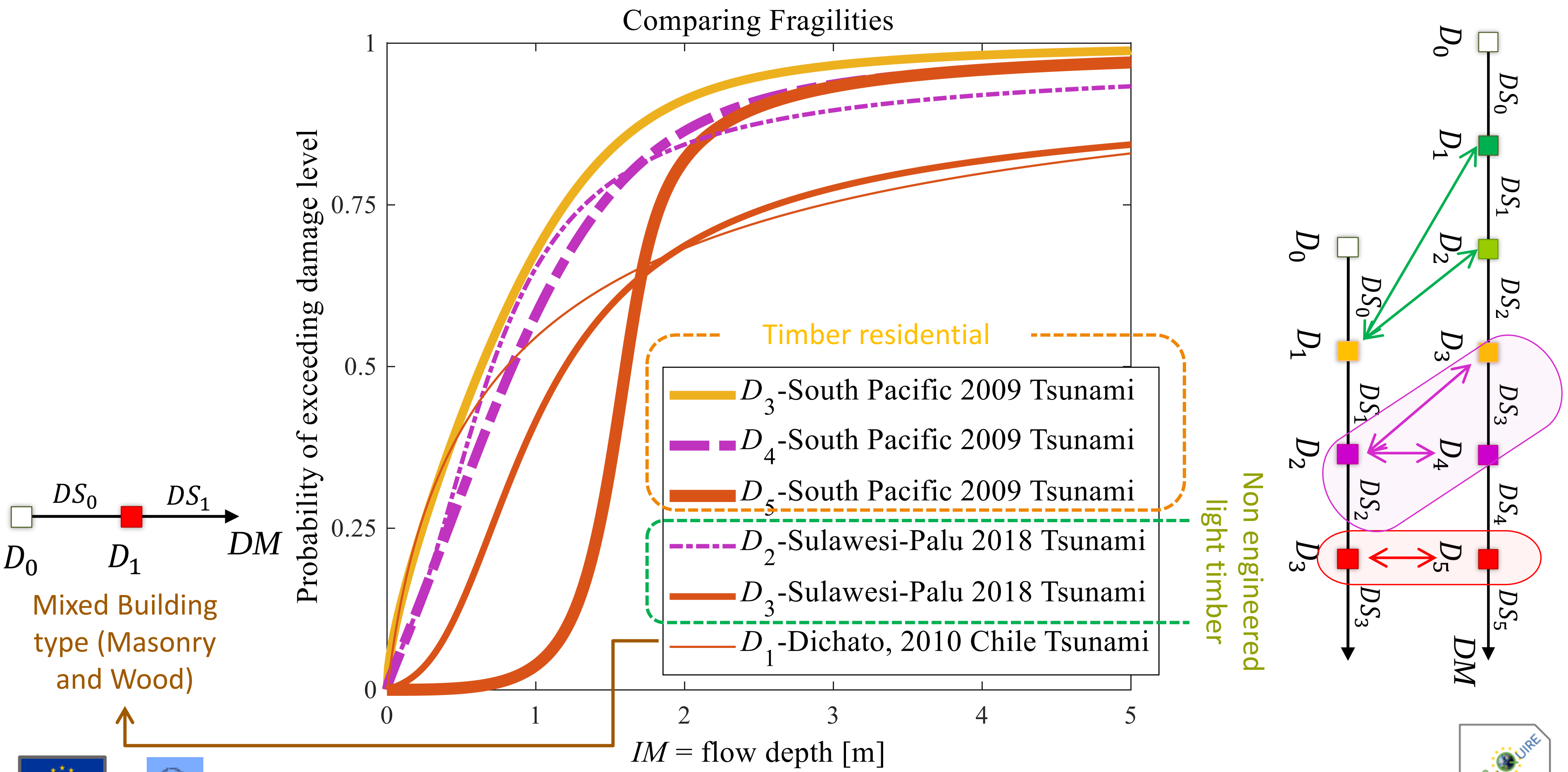
Damage Level		Damage level description
D_0	None	no damage
D_1	Light	non-structural damage
D_2	Minor	significant non-structural damage, minor structural damage
D_3	Moderate	significant structural and non-structural damage
D_4	Severe	irreparable structural damage, will require demolition
D_5	Collapse	complete structural collapse
		South Pacific 2009 **Reese et al. (2011)**

Damage Level		Damage level description
D_0	None	no damage
D_1	Repairable	Partial damage, repairable
D_2	Unrepairable	Partial damage, unrepairable
D_3	Complete	Complete structural collapse
		Sulawesi 2018 **Paulik et al. 2019**

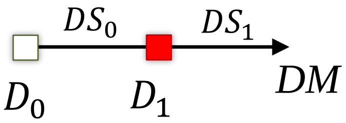
Reese, S., Bradley, B. A., Bind, J., Smart, G., Power, W., & Sturman, J. (2011). Empirical building fragilities from observed damage in the 2009 South Pacific tsunami. *Earth-Science Reviews*, *107*(1-2), 156-173.



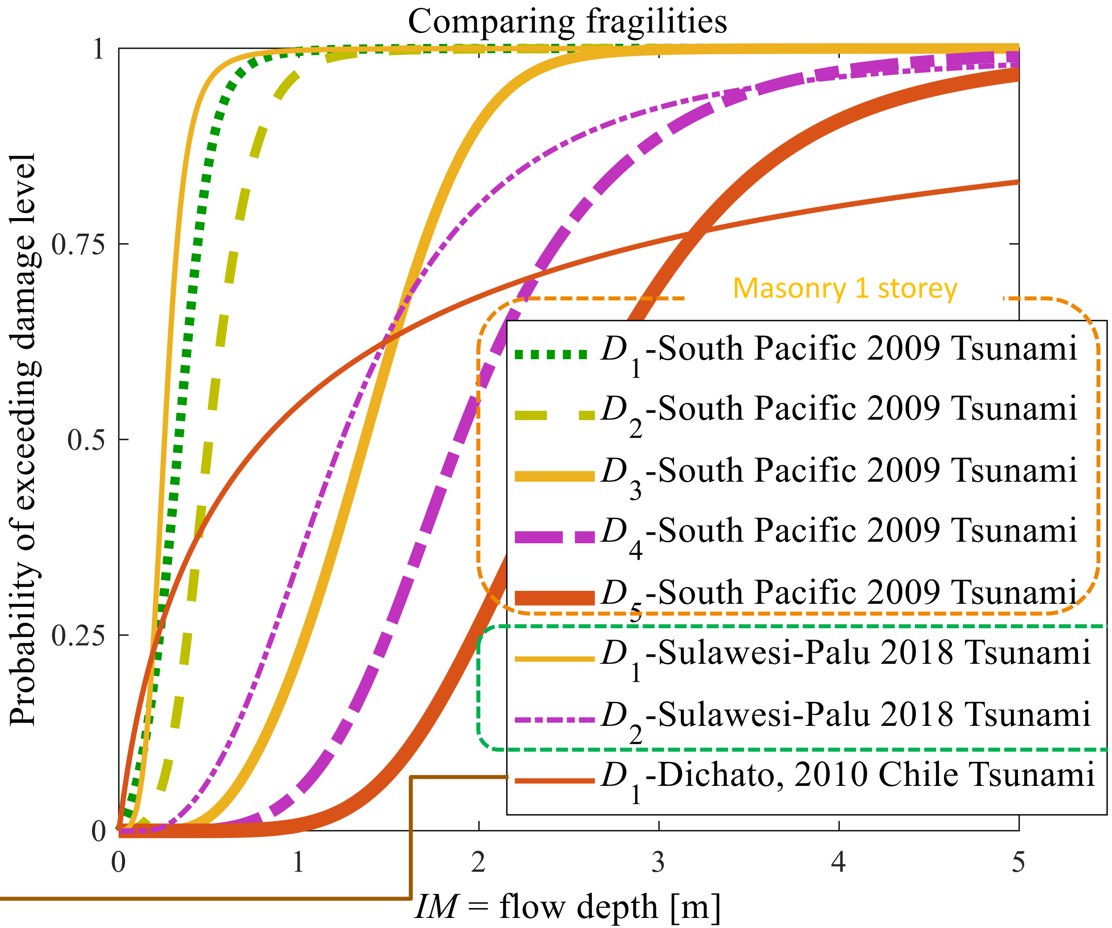
Paulik, R., Gusman, A., Williams, J. H., Pratama, G. M., Lin, S. L., Prawirabhakti, A., ... & Suwarni, N. W. I. (2019). Tsunami hazard and built environment damage observations from Palu City after the September 28 2018 Sulawesi earthquake and tsunami. *Pure and Applied Geophysics*, *176*(8), 3305-3321.

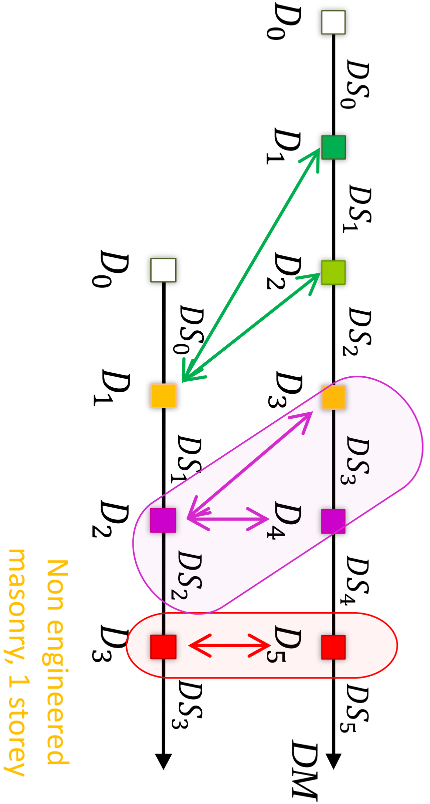

Comparing Fragilities
Probability of exceeding damage level
IM = flow depth [m]
Timber residential
D3-South Pacific 2009 Tsunami
D4-South Pacific 2009 Tsunami
D5-South Pacific 2009 Tsunami
D2-Sulawesi-Palu 2018 Tsunami
D3-Sulawesi-Palu 2018 Tsunami
D1-Dichato, 2010 Chile Tsunami
Non engineered light timber
Mixed Building type (Masonry and Wood)
D0 D1 DS0 DS1 DM
D0 D1 D2 D3 D4 D5 DS0 DS1 DS2 DS3 DS4 DS5 DM
D0 D1 D2 D3 DS0 DS1 DS2 DS3

Comparing fragility curves for similar classes of buildings



Mixed Building type (Masonry and Wood)





Vulnerability curves: propagating epistemic uncertainties in fragility

`calculate_vulnerability.m`

Vector of fragility model parameters

Sample fragility curve

$$G(DV > dv \mid im, \boldsymbol{\theta}) = \sum_{i=1}^{N_{DS}} G(DV > dv \mid DS_i) P(DS_i \mid im, \boldsymbol{\theta})$$

$P(DS_i \mid im, \boldsymbol{\theta})$

(Fragility Function)

DM ***(Damage Measure)***

e.g., damage states

$G_{DV|DS}(dv \mid DS_i)$

(Consequence Function)

DV ***(Decision Variable)***

e.g., fatalities, loss (loss ratio)

Sources of Uncertainty: fragility model parameters and consequence function

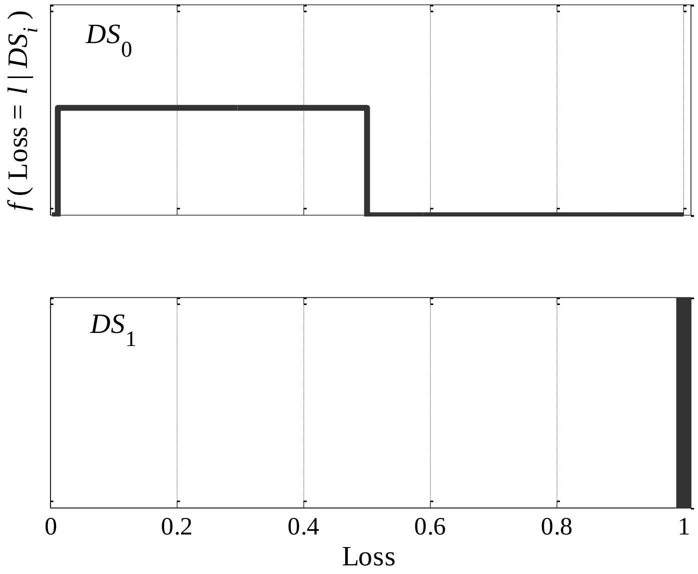
Vulnerability Curve with Epistemic Uncertainties

Median and logarithmic standard deviation of loss ratio given *IM*

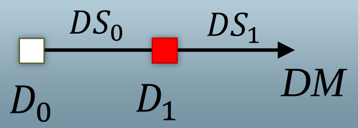
`Loss_modelk k=1,2,3`

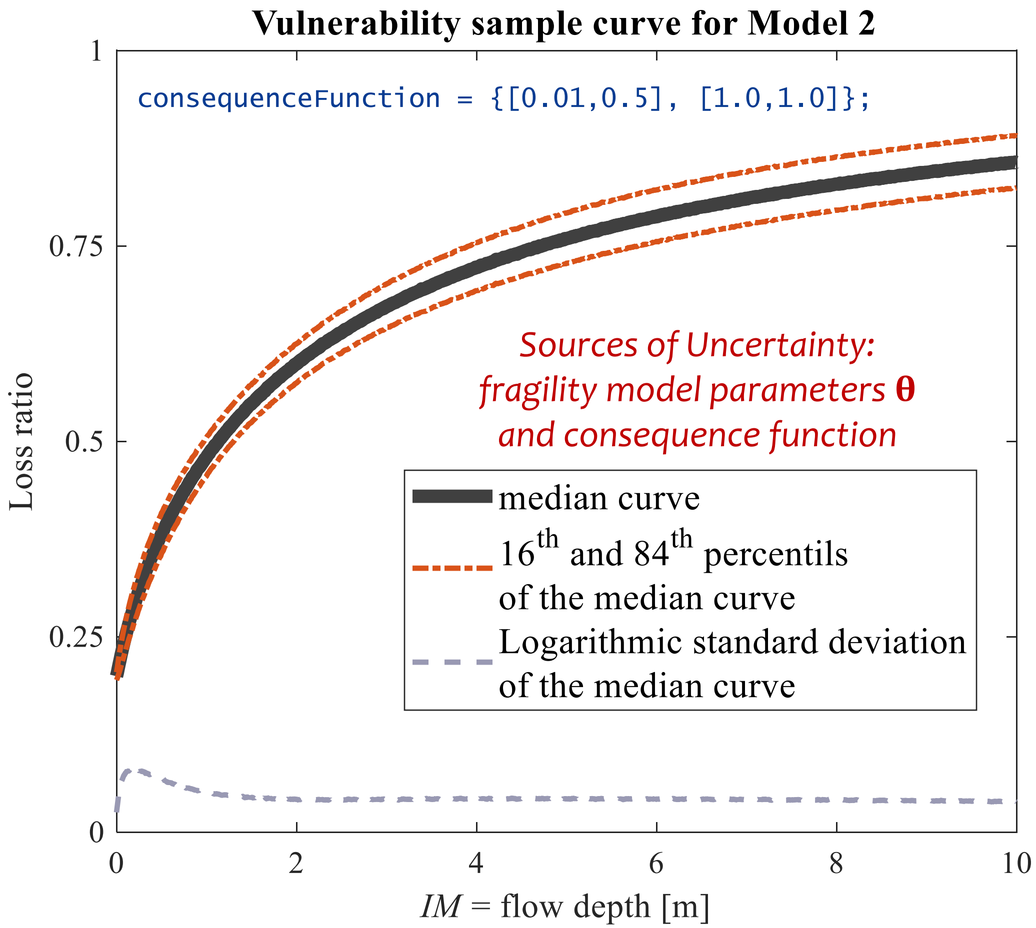
06 Tutorial

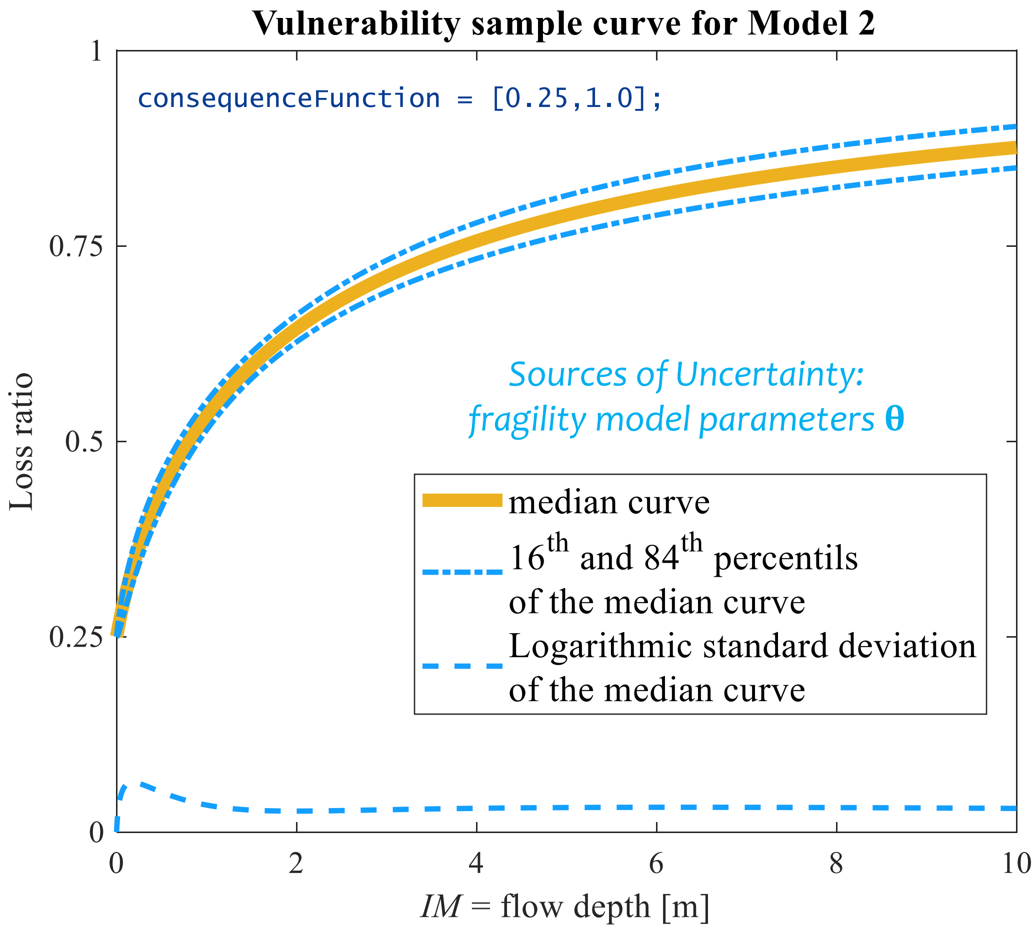
Vulnerability curves

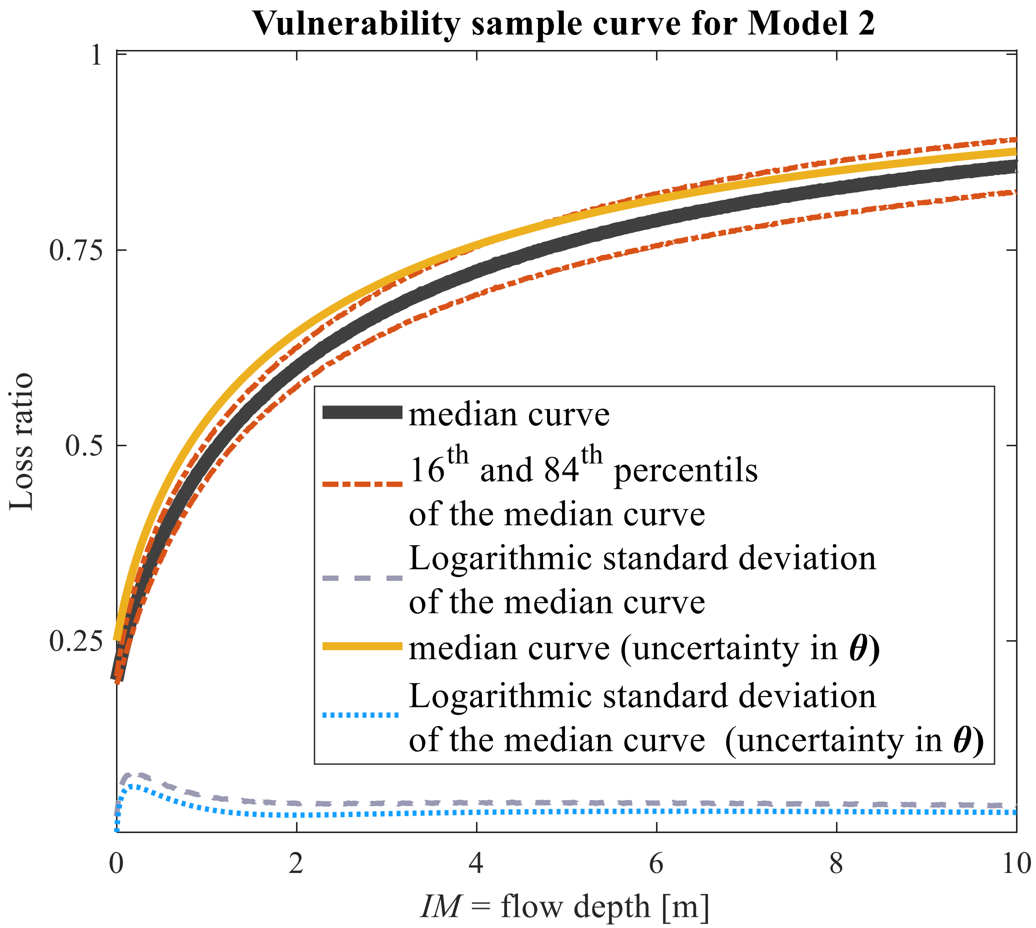


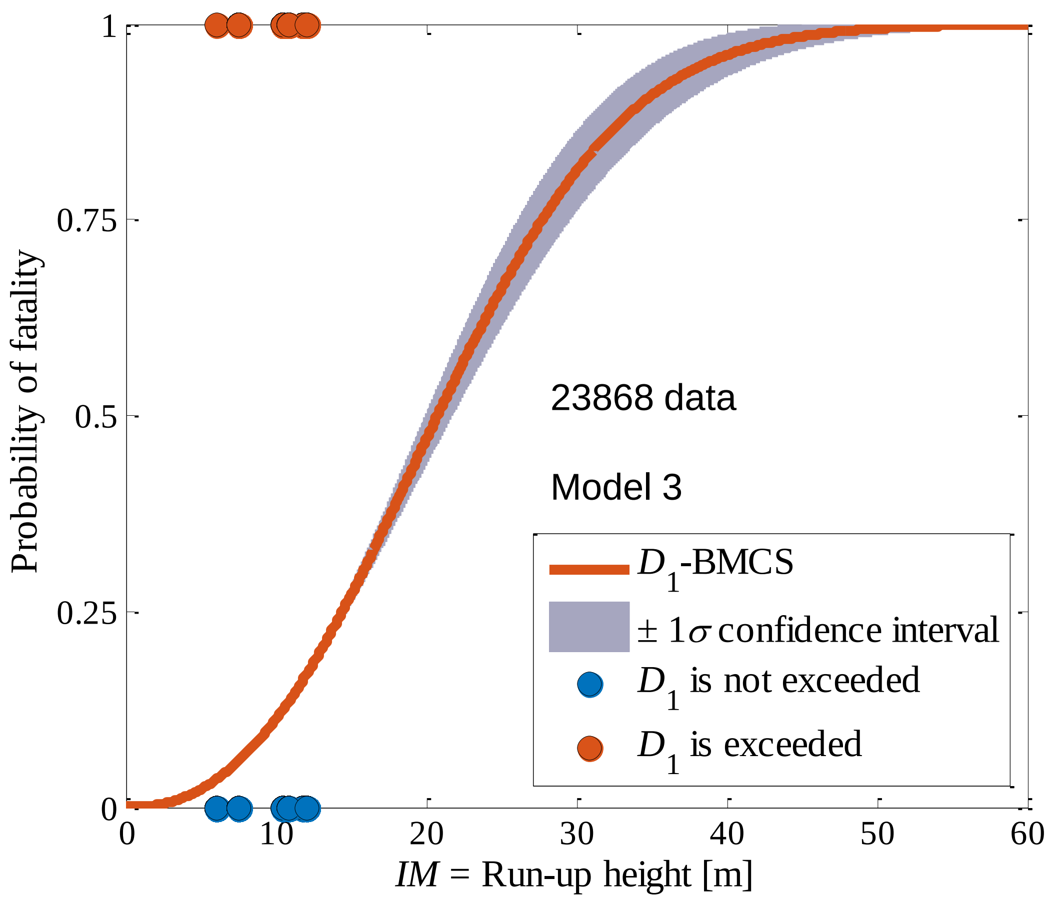
Mixed Building type (Masonry and Wood) of Dichato, Maule, 2010 Chile Tsunami




Vulnerability sample curve for Model 2
consequenceFunction = {[0.01,0.5], [1.0,1.0]};
Sources of Uncertainty:
fragility model parameters θ
and consequence function
median curve
16th and 84th percentiles of the median curve
Logarithmic standard deviation of the median curve
Loss ratio
0
0.25
0.5
0.75
1
0
2
4
6
8
10
IM = flow depth [m]


Vulnerability sample curve for Model 2
consequenceFunction = [0.25,1.0];
Sources of Uncertainty:
fragility model parameters θ
median curve
16th and 84th percentils of the median curve
Logarithmic standard deviation of the median curve
Loss ratio
0
0.25
0.5
0.75
1
0
2
4
6
8
10
IM = flow depth [m]


Vulnerability sample curve for Model 2
1
0.75
0.5
0.25
Loss ratio
0
2
4
6
8
10
IM = flow depth [m]
median curve
16th and 84th percentils
of the median curve
Logarithmic standard deviation
of the median curve
median curve (uncertainty in θ)
Logarithmic standard deviation
of the median curve (uncertainty in θ)



Santos, A., & Koshimura, S. (2015). The historical review of the 1755 Lisbon Tsunami. *J. Geodesy Geomat. Eng*, *1*, 38-52

Will be added to the ETRiS

`calculate_vulnerability.m`

`calculate_LossExceedance.m`

$$G(DV > dv \mid im, \boldsymbol{\theta}) = \sum_{i=1}^{N_{DS}} G(DV > dv \mid DS_i) P(DS_i \mid im, \boldsymbol{\theta})$$

$\lambda(IM > im \mid \varepsilon_{UH}(im))$

IM ***(Intensity Measure)***

e.g., flow depth, momentum flux

$G_{DV|DS}(dv \mid DS)$

(Fragility Function)

DM ***(Damage Measure)***

e.g., damage states

$P(DS_i \mid im, \boldsymbol{\theta})$

(Consequence Function)

DV ***(Decision Variable)***

e.g., fatalities, loss

$\lambda(DV > dv \mid \varepsilon_{UH}(im), \boldsymbol{\theta})$

$$\lambda(DV > dv \mid \varepsilon_{UH}(im), \boldsymbol{\theta}) = \int_{im} G_{DV|IM}(dv \mid im, \boldsymbol{\theta}) \left| d\lambda(IM > im \mid \varepsilon_{UH}(im)) \right|$$

Risk

e.g., AAL, LEC

Hazard Curve with Epistemic Uncertainties

Vulnerability Curve with Epistemic Uncertainties

Risk Curve

Behrens, J., Løvholt, F., Jalayer, F., Lorito, S., Salgado-Gálvez, M.A., Sørensen, M., Abadie, S., Aguirre-Ayerbe, I., Aniel-Quiroga, I., Babeyko, A. and Baiguera, M., 2021. Probabilistic tsunami hazard and risk analysis: A review of research gaps. *Frontiers in Earth Science*, *9*, p.628772.

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